Astronomy 11 — Fall 1998 (S.T. Myers)

Homework Set 4

Due Thursday October 29, 1998 (5pm)

1. Mars has only a very thin atmosphere, but is otherwise similar to the Earth. It is somewhat smaller, so maybe it couldn't hold a bigger atmosphere. Calculate the expected surface temperature of Mars, and then use this to find the minimum molecular weight a of a gas that can be retained over a billion years of time. Which gases would have escaped? Does this explain why Mars has such a thin atmosphere?

2. Observations from the Voyager mission show that Titan, the largest moon of Saturn, has an atmosphere! Use the condition for retention of a gas in the atmosphere over the course of a few billion years (the age of the planets) to find the minimum molecular weight a of a gas that could be in this atmosphere. The building blocks of simple amino acids are methane (CH_4), water (H_2O), carbon dioxide (CO_2), formaldehyde (H_2CO), and hydrogen cyanide (HCN). Compute the molecular weights of these molecules and determine whether they would be retained on Titan. (You can find the physical parameters of Titan in the appendix of the text, and you may have to make a guess on some other properties — be sure to justify these.)

Ganymede, the satellite of Jupiter, is the largest moon in our solar system. Would you expect it to have an atmosphere?

3. The angular momentum of a spherical rotating body is given by $L_{rot} = I\,\omega$ where I is the moment of inertia and ω is the angular frequency in radians/sec ($\omega = 2\pi/P$). For a sphere, $I = \frac{2}{5}MR^2$. The orbital angular momentum of a two body system with semimajor axis a and eccentricity e we had showed to be

$$L_{orb} = \mu\sqrt{GMa(1-e^2)} \quad \rightarrow \quad L_{orb} = \mu\sqrt{GMa} = \mu\omega a^2$$

where we have assumed a nearly circular orbit $e = 0$. Remember that the reduced mass $\mu = m_1 m_2/M$, $M = m_1 + m_2$. A third relation of use is Kepler's Third Law

$$GMP^2 = 4\pi^2 a^3 \quad \rightarrow \quad \omega^2 = \frac{4\pi^2}{P^2} = \frac{GM}{a^3}$$

which we used partly to arrive at the final expression for L_{orb} above.

(a) Compute the rotational angular momenta L_{rot} of the Earth and the Moon (get the masses, radii, and rotational periods from the textbook). Then, compute the orbital angular momentum L_{orb} of the Earth-Moon system. Compare the two — which component dominates the angular momentum of the Earth-Moon system now? Since the rotational and orbital axes are approximately aligned, sum the three to get the total angular momentum of the Earth-Moon system $L_{tot} = L_{rot} + L_{orb}$.

(b) The tidal locking effect will eventually increase the Earth's rotational period and lengthen the Moon's rotational and orbital periods so that all three are equal. To compute the evolution of the orbit and rotation, we need to arrive at an expression for L_{tot} which only depends on one of the variables a and ω. Use Kepler's 3rd Law above to substitute in for a in L_{orb} so you have an expression that only depends on ω, and thus derive an equation for L_{tot} (which will have three terms for the rotation of the Earth, rotation of the moon, and the relative orbit) as a function of ω.

(c) Use conservation of the total angular momentum and the current total angular momentum from part (a), and the equation you derived in part (b), to find the final equilibrium value of ω_{eq} and thus the period P_{eq}. (Hint: if you have trouble solving the equation, just use the dominant term and assume all of the initial angular momentum is transferred into that. You should get a final period in the neighborhood of 50 days.)