**Astronomy 11 — Fall 1998 (S.T. Myers)**

**Homework Set 2**

Due Thursday October 1, 1998 (5pm)

**1.** Local noon (12h local solar time) is defined to occur when the Sun is on the observer's local meridian. On approximately what day of the year does local noon coincide with 12h local sidereal time (LST)? One month later, what is the approximate LST at local noon? Use these numbers to work out approximate what the LST is at local noon on your birthday.

**2.** Deimos is a satellite of Mars with an orbital eccentricity of $e = 0.003$ (and thus in a nearly circular orbit) and semimajor axis $a = 23520$ km and period $P = 1.26$ days. Using Newton's Laws to derive Kepler's Third Law for a circular orbit we found

$$GMP^2 = 4\pi^2 a^3$$

Use this to determine the mass of Mars (in kg, and in relation to Earth's mass). (Remember in this equation that MKS units are assumed. You can find $G$ in Appendix Table A7-2 in the text book, or in the online constant table.

**3.** The *angular velocity* $\dot{\theta} = d\theta/dt$ of a planet at some point in its orbit is given by $\dot{\theta} = v_\theta/r$ where $v_\theta$ is the component of the orbital velocity perpendicular to the orbital radius vector. Note that this angular velocity also represents the apparent motion of the Sun against the ecliptic as seen from the planet. We had found in class that the *angular momentum per unit mass* $H = v_\theta r$ is conserved in the orbit. Mars has an eccentricity $e = 0.093$ and so the variations in the Sun's apparent will be more pronounced than that of Earth. Find the ratio of eastward apparent angular velocities of the Sun at perihelion and aphelion $\dot{\theta}_p/\dot{\theta}_a$ as seen from Mars. Also, compute the average eastward motion of the Sun from Mars (in degrees per day).

**4.** The circular velocity $v_{circ}$ at distance $r$ from the Sun (or any other body) is the velocity of a circular orbit with radius $a = r$

$$v_{circ} = \frac{2\pi a}{P} = \sqrt{\frac{GM_\odot}{a}}$$

which we derived using the centripetal and gravitational forces. Calculate the circular velocities for Earth's orbit (1 AU) and Mars's orbit (1.524 AU). Pretend the orbits of Mars and Earth are circular. There is an elliptical orbit that has perihelion at Earth's orbit and aphelion at Mars's orbit — this is the *least energy orbit* along which a space probe can be sent from Earth to Mars. Find the semimajor axis $a$ and eccentricity $e$ of this orbit.

Then, calculate the perihelion velocity $v_p$ of this orbit and compare this to the circular velocity at Earth's orbit to find the extra velocity $v_{inj}$ needed to be given to a probe to put it on a path to Mars. Assume that the probe is moving along with the Earth in its circular orbit around the Sun (ignore its orbital velocity around the Earth).

Finally, compute the aphelion velocity $v_a$ of this orbit and compare this to the circular velocity of Mars's orbit to find the $v_{ret}$ needed to place the probe into circular orbit with Mars. Should this velocity be added or removed from the aphelion velocity? (Ignore its orbital velocity around Mars, condider only solar orbits. You can get expressions for the perihelion and aphelion velocities by using conservation of angular momentum $H = v_p r_p = v_a r_a$, the fact that $v_{circ}^2 = v_p v_a$, and the properties of ellipses.)