Astronomy 11 Fall 1998 (S.T. Myers)

Equations for Final Exam   Tue 15 December 1998

Constants

$c = 2.998 \times 10^5$ km/s

$\mathrm{h} = 6.626 \times 10^{-34}\,\mathrm{J\,s}$

$\mathrm{k} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$\mathrm{e} = 1.602 \times 10^{-19}\,\mathrm{C}$

$\mathrm{m_p} = 1.67 \times 10^{-27}\,\mathrm{kg}$

$M_\odot = 2 \times 10^{30}$ kg

$L_\odot = 3.83 \times 10^{26}$ W

$M_\oplus = 5.98 \times 10^{24}$ kg

$g_\oplus = 9.807$ m/s^2

$1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$

$1\,\mathrm{radian} = 206265''$

$\mathrm{G} = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

$1\mathrm{\AA} = 10^{10}$ m $= 10^{-4}\,\mu$m

$1\,\mathrm{eV} = 1.602 \times 10^{-19}$ J

$\mathrm{m_e} = 9.11 \times 10^{-31}\,\mathrm{kg}$

$R_\odot = 6.96 \times 10^5$ km

$T_\odot = 5770$ K

$R_\oplus = 6378$ km

Solar Constant = 1370 W/m^2

$1\,\mathrm{pc} = 206265\,\mathrm{AU} = 3.086 \times 10^{16}$ m

1 year = 3.156×10^7 seconds = 365.26 days

Mechanics

$$\vec{v} = d\vec{r}/dt \qquad \vec{a} = d\vec{v}/dt$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = \vec{r} \times \vec{p}$$

$$F_{cen} = \frac{mv^2}{r} \qquad F = \frac{Gm_1m_2}{r^2}$$

$$E = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} \qquad \frac{1}{S} = \frac{1}{P_{in}} - \frac{1}{P_{out}}$$

$$v_{circ} = \frac{2\pi a}{P} \qquad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$M = m_1 + m_2 \qquad GM\,P^2 = 4\pi^2\,a^3$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 = 0 \qquad M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\vec{r}_1 = -\frac{\mu}{m_1}\vec{r} \qquad \vec{r}_2 = \frac{\mu}{m_2}\vec{r}$$

$$m_1r_1 = m_2r_2 \qquad m_1v_1 = m_2v_2$$

$$r_p = a(1-e) \qquad r_a = a(1+e)$$

$$H = rv_\theta \qquad L_{orb} = \mu v_p r_p = \mu v_a r_a$$

$$\frac{E}{\mu} = -\frac{GM}{2a} \qquad H^2 = GMa(1-e^2)$$

$$v_{circ}^2 = \frac{GM}{r} \qquad v_{esc}^2 = \frac{2GM}{r}$$

$$v_\theta^2 = \frac{GMa}{r^2}(1-e^2) \qquad v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

$$\vec{F} \cdot d\vec{r} = F_r dr + F_\theta r d\theta \qquad d\vec{r}/dt = (dr/dt, r d\theta/dt)$$

Electromagnetism and Optics:

$$\nu = c/\lambda$$
$$F = L/4\pi r^2$$
$$\lambda_{max} = \frac{2.9\,\mathrm{mm}}{(T/\,1\mathrm{K})}$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$y = f\,\theta$$
$$F_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$
$$F_{mag} = q\vec{v}\times\vec{B}$$

$$E = h\nu = hc/\lambda$$
$$F_{surf} = \sigma T^4$$
$$\theta_{res} = \frac{\lambda}{D}$$
$$n = c/v \geq 1$$
$$P_{in} = F_{in}\cdot \pi D^2/4$$
$$\frac{1}{4\pi\epsilon_0} = 9\times 10^9\ \mathrm{Nm^2C^{-2}}$$
$$B(r) = B_0\left(\frac{r}{r_0}\right)^{-3}$$

Planetary Astrophysics

$$P_{abs} = \pi R^2\, F_{in}\,(1-A)$$
$$P_{ref} = \pi R^2\, F_{in}\, A$$
$$T_{eq} = \frac{T_{ss}}{\sqrt{2}} \quad \text{(sphere)}$$
$$p = nkT = \frac{\rho k T}{a m_H}$$
$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} = -g\rho$$
$$U \approx \frac{GM^2}{R}$$
$$p = p_0\, e^{-h/H}$$

$$P_{em} = 4\pi R^2\,\sigma\, T^4\,(1-G)$$
$$F_{in} = \sigma\, T_{ss}^4 = L/4\pi r^2$$
$$T_{surf} = T_{eq}\left[\frac{1-A}{1-G}\right]^{1/4}$$
$$v_{rms}^2 = \frac{3kT}{m}$$
$$P_c \sim \frac{GM\bar{\rho}}{R}$$
$$T_c \sim \frac{\mu m_H}{k}\,\frac{P_c}{\bar{\rho}}$$
$$H = \frac{kT}{mg}$$

Common Elements and Isotopes:

(Key: ${}^{mass}_{num}X$)	${}^{1}_{1}\mathrm{H}$	${}^{4}_{2}\mathrm{He}$	${}^{12}_{6}\mathrm{C}$	${}^{14}_{7}\mathrm{N}$
	${}^{16}_{8}\mathrm{O}$	${}^{20}_{10}\mathrm{Ne}$	${}^{32}_{16}\mathrm{S}$	${}^{56}_{26}\mathrm{Fe}$

Mass of ${}^{1}\mathrm{H} = 1.007825$ amu $= 1.67352\times 10^{-27}$ kg

Mass of ${}^{4}\mathrm{He} = 4.00268$ amu

Mass of ${}^{12}\mathrm{C} \equiv 12$ amu

Representative Mean Densities:

	Water	Gas	Rock	Metal
$\bar{\rho}$ **(kg/m^3):**	1000	500 – 1700	∼3500	8000 – 10000

Solar System Data:

Planet	a **(AU)**	P **(days)**	e	$M/M_\oplus$	$R/R_\oplus$	**Albedo**
Mercury	0.387	87.96	0.206	0.056	0.38	0.06
Venus	0.723	224.70	0.007	0.815	0.95	0.76
Earth	1.0	365.26	0.017	1.0	1.0	0.40
Mars	1.524	686.98	0.093	0.107	0.53	0.16
Jupiter	5.203	4333	0.048	318	11.19	0.51
Saturn	9.54	10759	0.056	95	9.41	0.50
Uranus	19.18	30685	0.047	14.5	4.01	0.66
Neptune	30.06	60188	0.009	17.2	3.89	0.62
Pluto	39.44	90700	0.249	0.0018	0.18	0.50