Polarization in Interferometry

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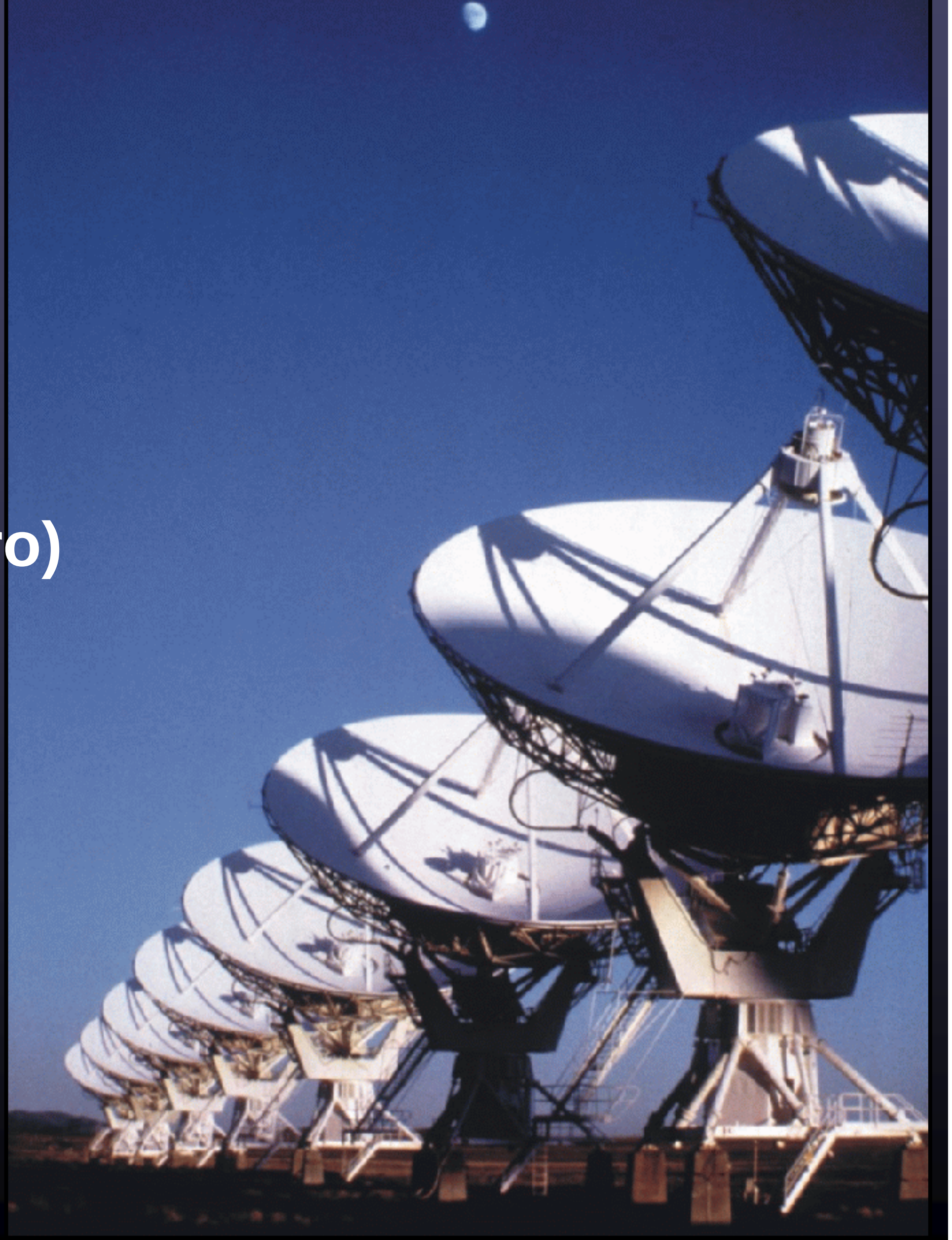


Eleventh Synthesis Imaging Workshop

Socorro, June 10-17, 2008

Polarization in interferometry

- Astrophysics of Polarization
- Physics of Polarization
- Antenna Response to Polarization
- Interferometer Response to Polarization
- Polarization Calibration & Observational Strategies
- Polarization Data & Image Analysis

WARNING!

- This is tough stuff. Difficult concepts, hard to explain without complex mathematics.
- I will illustrate the concepts with figures and ‘handwaving’.
- Many good references:
  - Synthesis Imaging II: Lecture 6, also parts of 1, 3, 5, 32
  - Born and Wolf: Principle of Optics, Chapters 1 and 10
  - Rolfs and Wilson: Tools of Radio Astronomy, Chapter 2
  - Thompson, Moran and Swenson: Interferometry and Synthesis in Radio Astronomy, Chapter 4
  - Tinbergen: Astronomical Polarimetry. All Chapters.
  - J.P. Hamaker et al., A&A, 117, 137 (1996) and series of papers
- Great care must be taken in studying these – conventions vary between them.

DON’T PANIC !

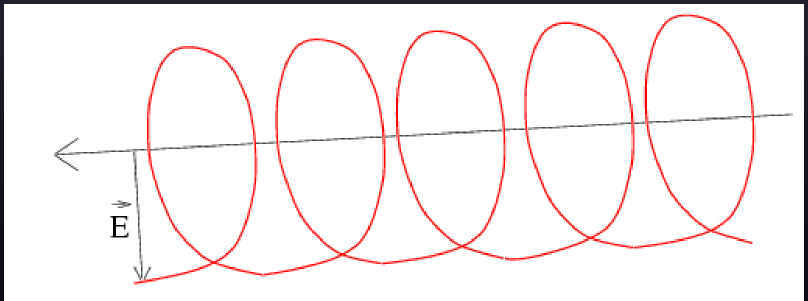
Polarization Astrophysics

What is Polarization?

- Electromagnetic field is a vector phenomenon – it has both direction and magnitude.
- From Maxwell's equations, we know a propagating EM wave (in the far field) has no component in the direction of propagation – it is a transverse wave.

$$\mathbf{k} \bullet \mathbf{E} = 0$$

- The characteristics of the transverse component of the electric field, E, are referred to as the polarization properties. The E-vector follows a (elliptical) helical path as it propagates:



Why Measure Polarization?

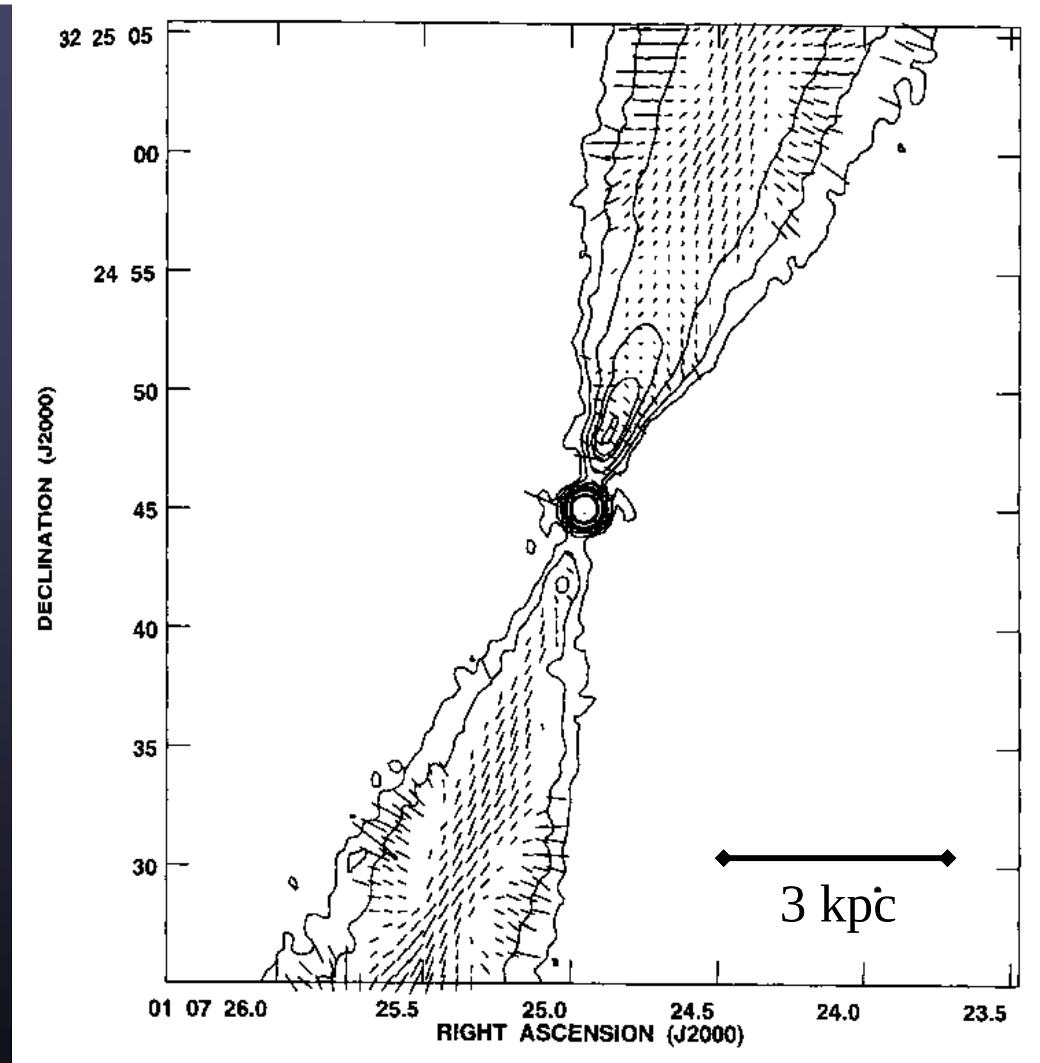
- Electromagnetic waves are intrinsically polarized
  - monochromatic waves are fully polarized
- Polarization state of radiation can tell us about:
  - the origin of the radiation
    - intrinsic polarization
  - the medium through which it traverses
    - propagation and scattering effects
  - unfortunately, also about the purity of our optics
    - you may be forced to observe polarization even if you do not want to!

Astrophysical Polarization

- Examples:
  - Processes which generate polarized radiation:
    - Synchrotron emission: Up to ~80% linearly polarized, with no circular polarization. Measurement provides information on strength and orientation of magnetic fields, level of turbulence.
    - Zeeman line splitting: Presence of B-field splits RCP and LCP components of spectral lines by by 2.8 Hz/μG. Measurement provides direct measure of B-field.
  - Processes which modify polarization state:
    - Free electron scattering: Induces a linear polarization which can indicate the origin of the scattered radiation.
    - Faraday rotation: Magnetoionic region rotates plane of linear polarization. Measurement of rotation gives B-field estimate.
    - Faraday conversion: Particles in magnetic fields can cause the polarization ellipticity to change, turning a fraction of the linear polarization into circular (possibly seen in cores of AGN)

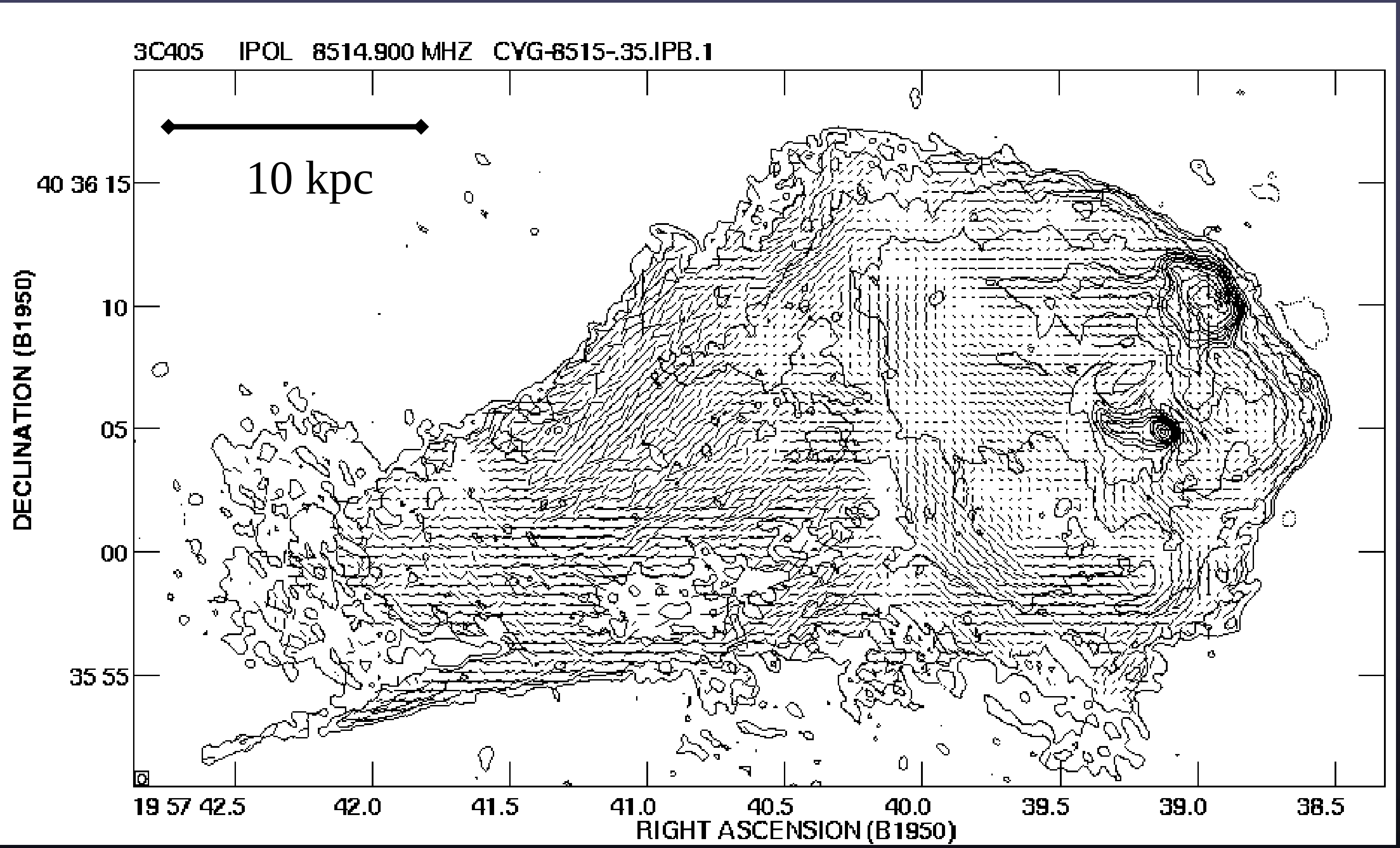
Example: Radio Galaxy 3C31

- VLA @ 8.4 GHz
- E-vectors
  - along core of jet
  - radial to jet at edge
- Laing (1996)



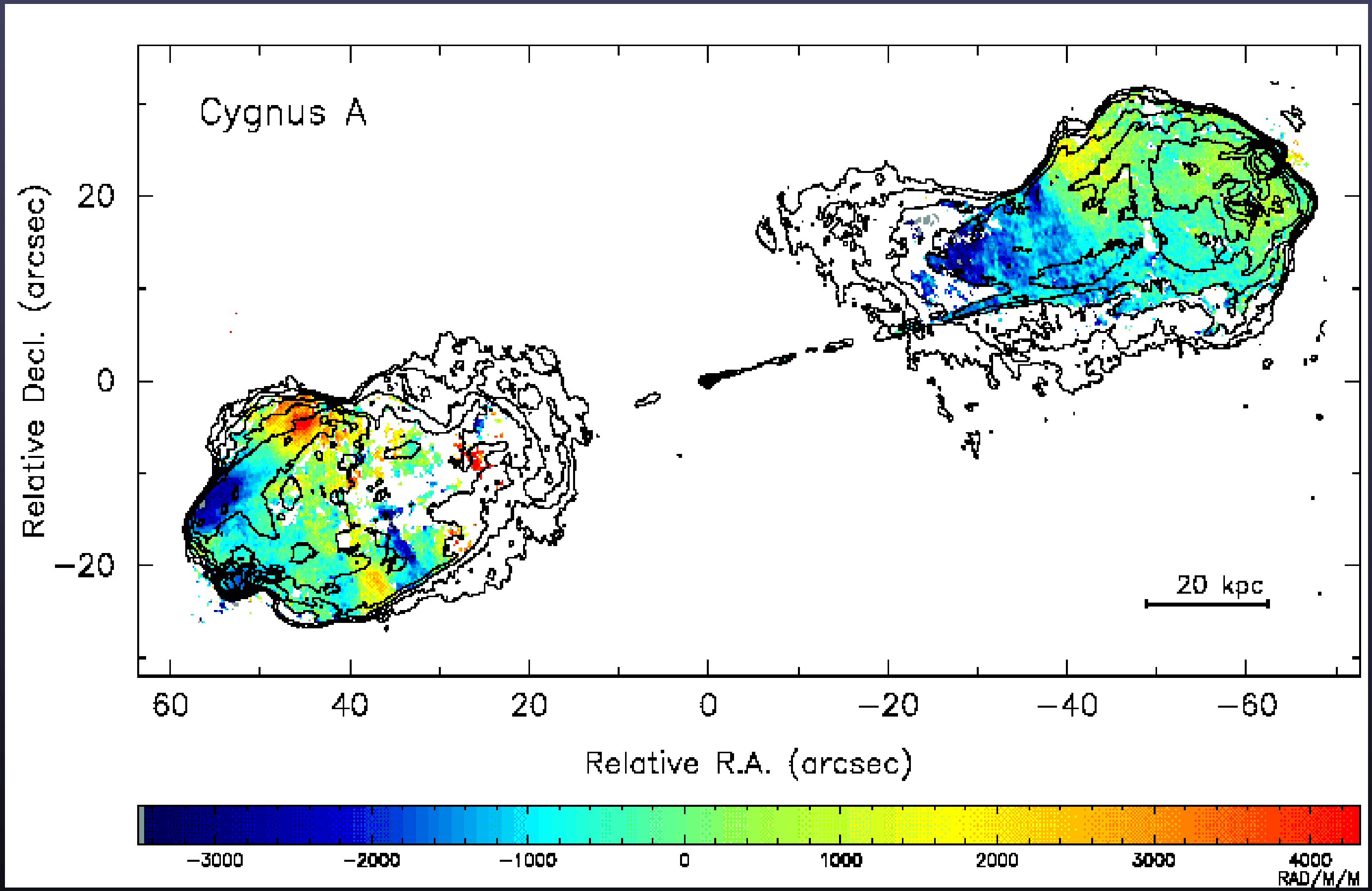
Example: Radio Galaxy Cygnus A

- VLA @ 8.5 GHz   B-vectors   Perley & Carilli (1996)



Example: Faraday rotation of CygA

– See review of “Cluster Magnetic Fields” by Carilli & Taylor 2002 (ARAA)



Example: Zeeman effect

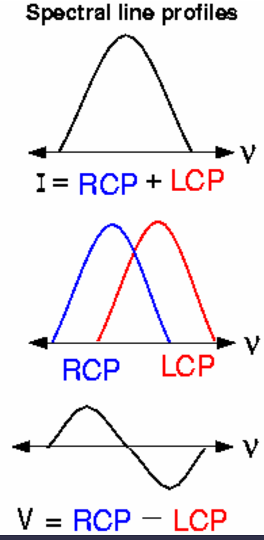
Zeeman Effect

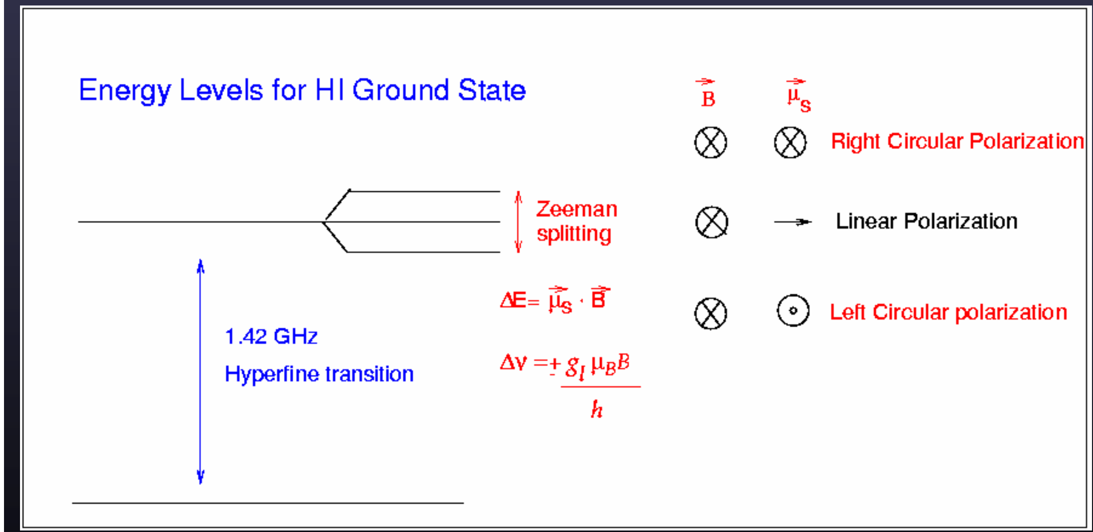
Atoms and molecules with a net magnetic moment will have their energy levels split in the presence of a magnetic field.

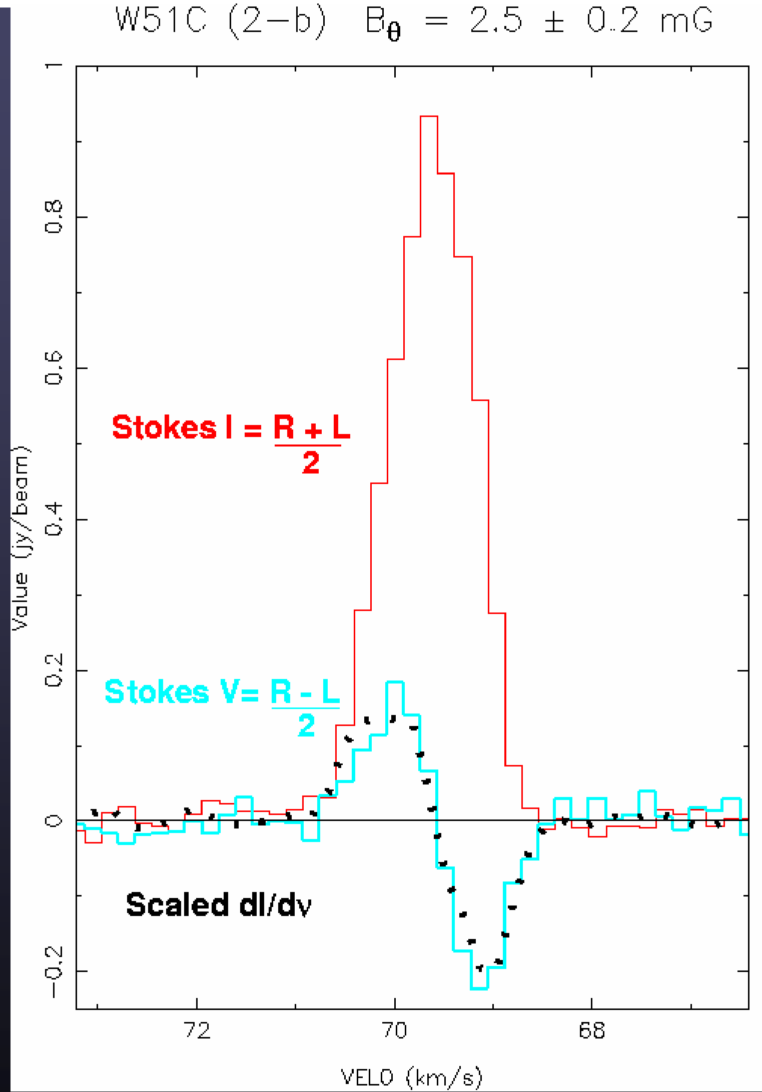
⇒ HI, OH, CN, H_2O

⇒ Detected by observing the frequency shift between right and left circularly polarized emission

⇒ V = RCP − LCP $\propto B_{los}$

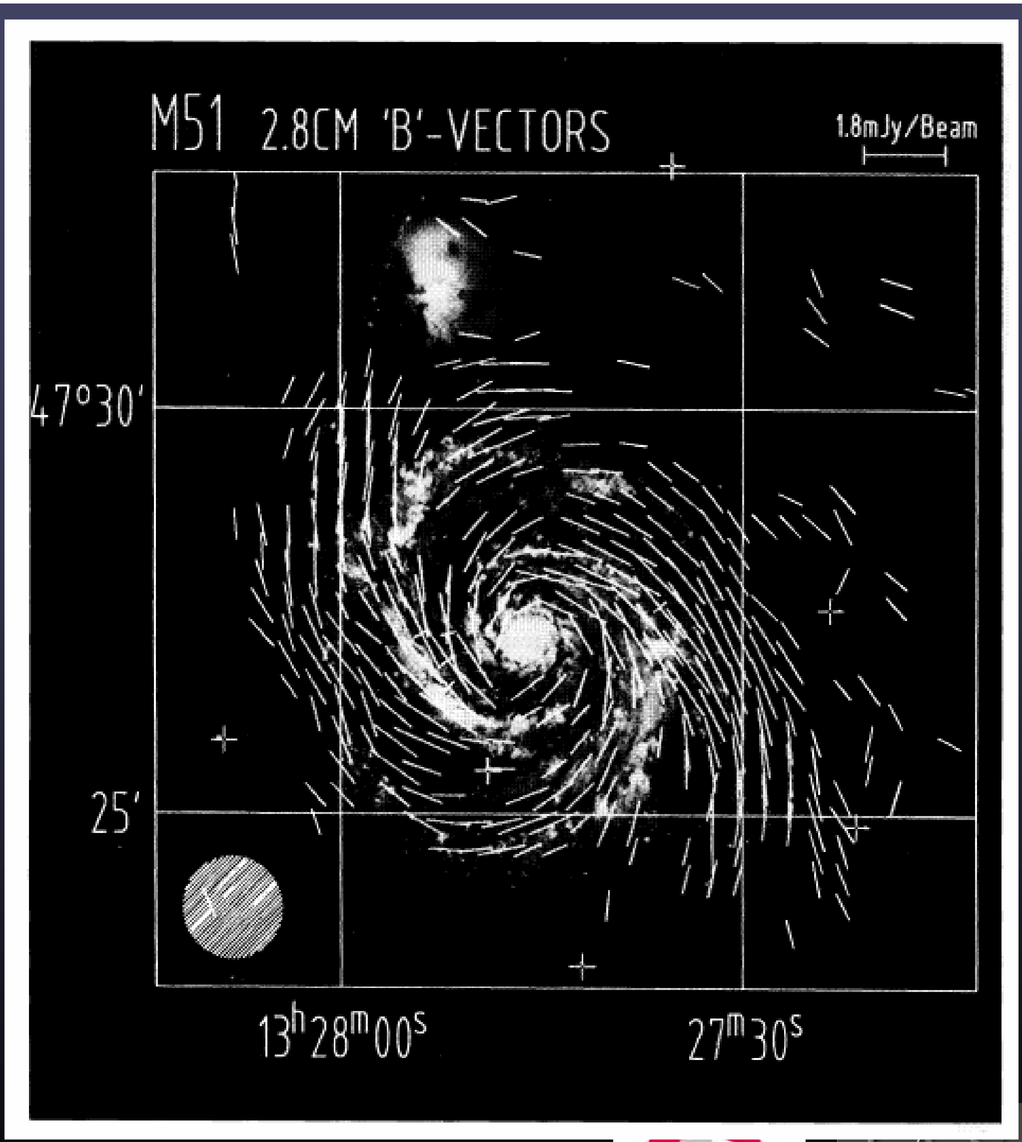






Example: the ISM of M51

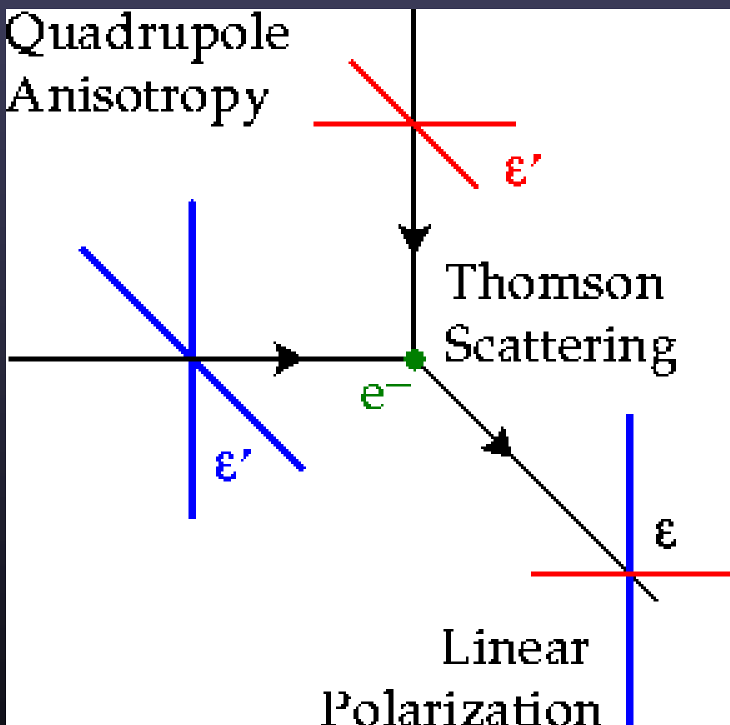
- Trace magnetic field structure in galaxies



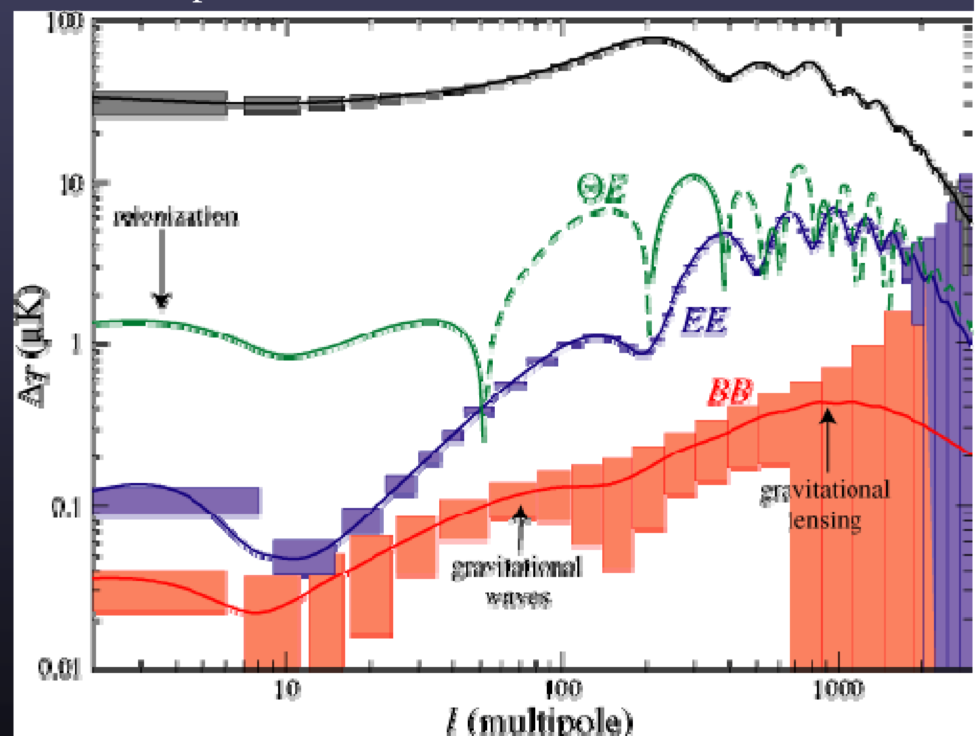
Neininger (1992)

Scattering

- Anisotropic Scattering induces Linear Polarization
  - electron scattering (e.g. in Cosmic Microwave Background)
  - dust scattering (e.g. in the millimeter-wave spectrum)



Planck predictions – Hu & Dodelson *ARAA 2002*



Animations from Wayne Hu

Polarization Fundamentals

The Polarization Ellipse

- From Maxwell's equations E•B=0 (E and B perpendicular)
  - By convention, we consider the time behavior of the E-field in a fixed perpendicular plane, from the point of view of the receiver.
- For a monochromatic wave of frequency ν, we write

$$E_x = A_x \cos(2\pi\upsilon\, t + \phi_x)$$
$$E_y = A_y \cos(2\pi\upsilon\, t + \phi_y)$$

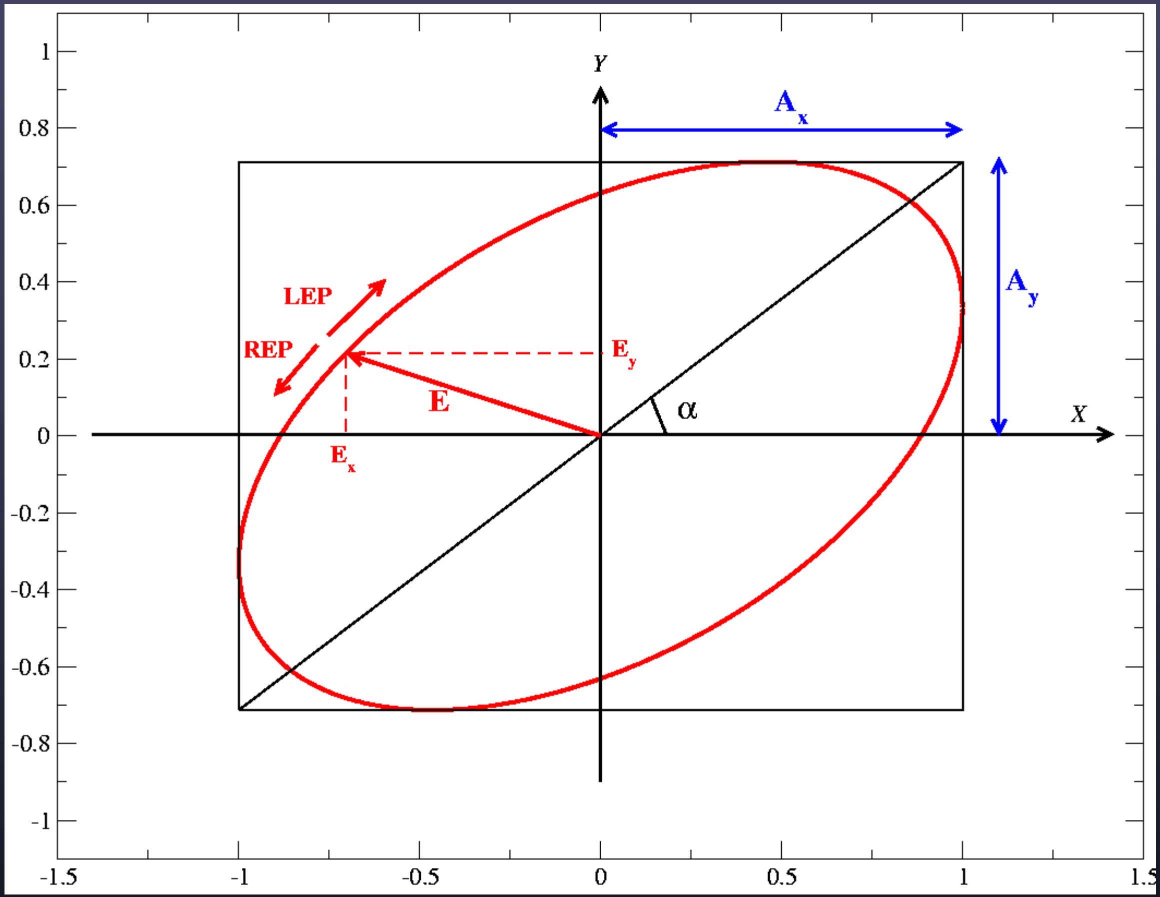
  - These two equations describe an ellipse in the (x-y) plane.
- The ellipse is described fully by three parameters:
  - A_X, A_Y, and the phase difference, $\delta = \phi_Y - \phi_X$.
- The wave is elliptically polarized. If the E-vector is:
  - Rotating clockwise, the wave is 'Left Elliptically Polarized',
  - Rotating counterclockwise, it is 'Right Elliptically Polarized'.

Elliptically Polarized Monochromatic Wave

The simplest description of wave polarization is in a Cartesian coordinate frame.

In general, three parameters are needed to describe the ellipse.

The angle $\alpha = \mathrm{atan}(A_Y/A_X)$ is used later …



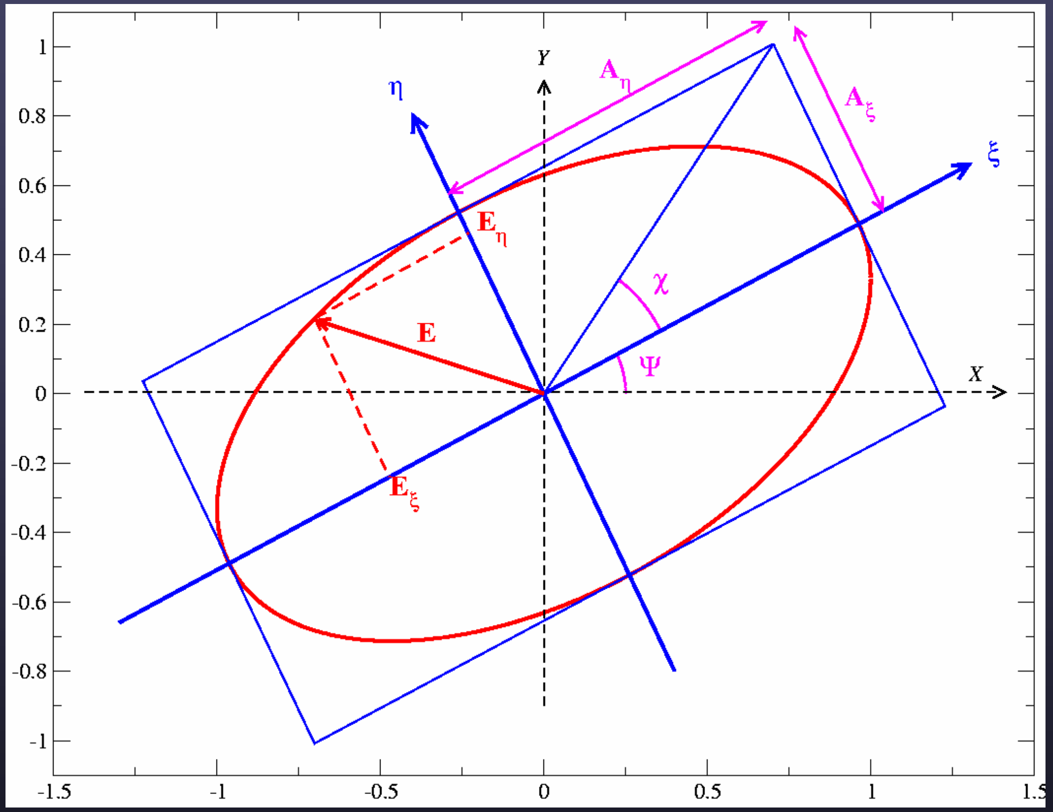
Polarization Ellipse Ellipticity and P.A.

- A more natural description is in a frame (ξ,η), rotated so the ξ-axis lies along the major axis of the ellipse.
- The three parameters of the ellipse are then:
  - A_η : the major axis length
  - $\tan\chi = A_\xi/A_\eta$: the axial ratio
  - Ψ : the major axis p.a.

$$\tan 2\Psi = \tan 2\alpha \cos\delta$$

$$\sin 2\chi = \sin 2\alpha \sin\delta$$

- The ellipticity χ is signed:
  - $\chi > 0 \rightarrow$ REP
  - $\chi < 0 \rightarrow$ LEP



χ = 0,90° $\rightarrow$ Linear (δ=0°,180°)

χ = ±45° $\rightarrow$ Circular (δ=±90°)

Circular Basis

- We can decompose the E-field into a circular basis, rather than a (linear) Cartesian one:

$$\mathbf{E} = A_R \hat{e}_R + A_L \hat{e}_L$$

  – where A_R and A_L are the amplitudes of two counter-rotating unit vectors, e_R (rotating counter-clockwise), and e_L (clockwise)

  – NOTE: R,L are obtained from X,Y by $\delta=\pm 90°$ phase shift

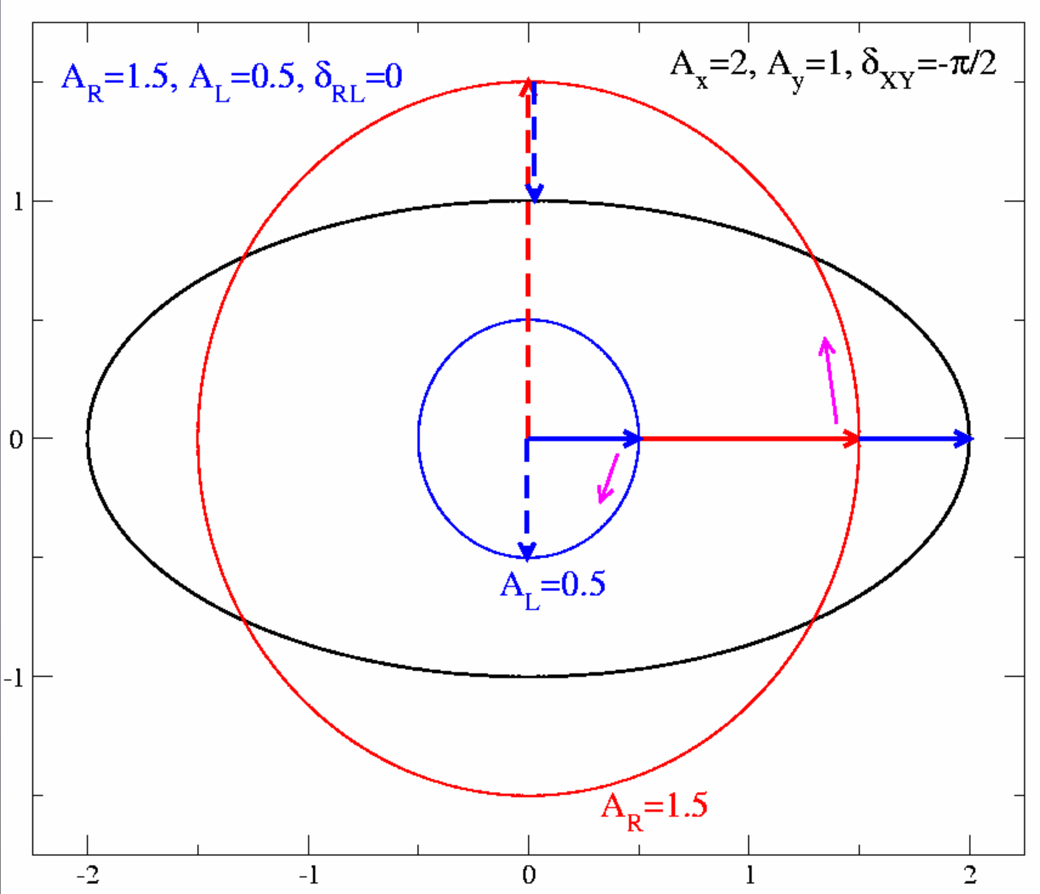
- It is straightforward to show that:

$$A_R = \frac{1}{2}\sqrt{A_X^2 + A_Y^2 - 2\,A_X A_Y \sin\delta_{XY}}$$

$$A_L = \frac{1}{2}\sqrt{A_X^2 + A_Y^2 + 2\,A_X A_Y \sin\delta_{XY}}$$

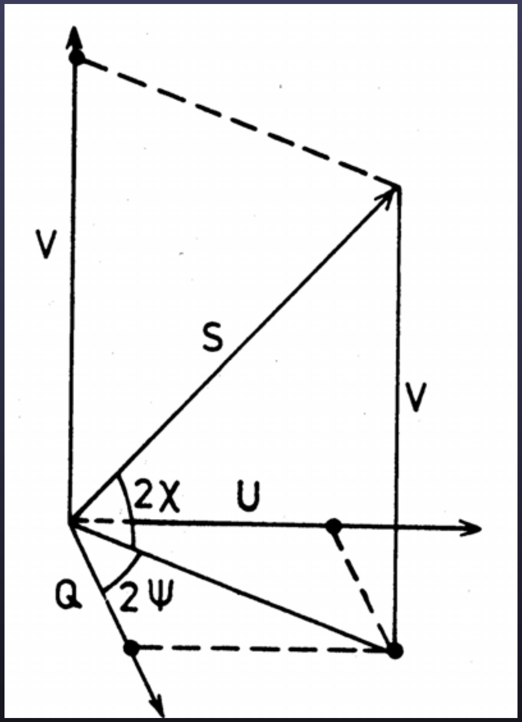
Circular Basis Example

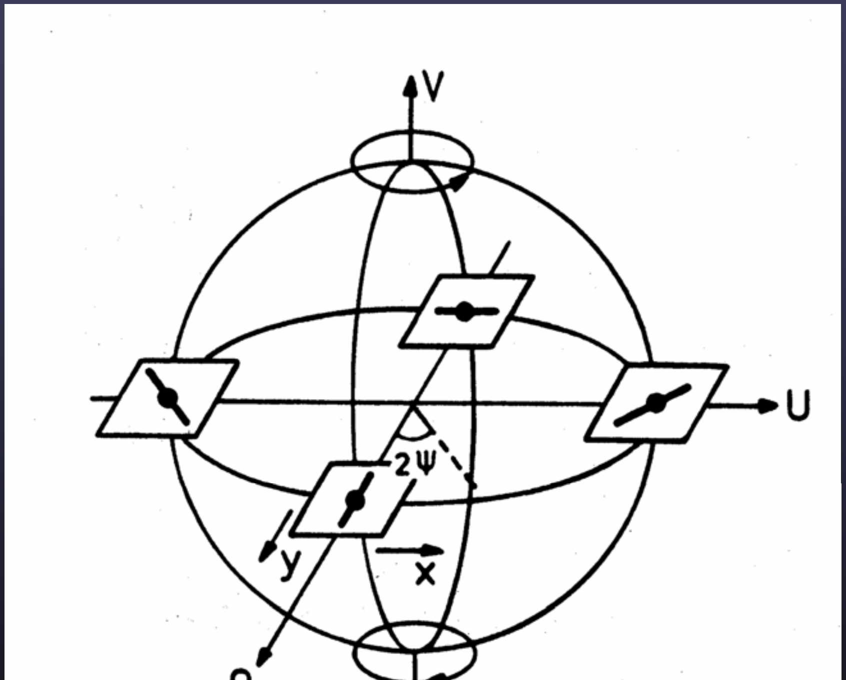
- The black ellipse can be decomposed into an x-component of amplitude 2, and a y-component of amplitude 1 which lags by ¼ turn.
- It can alternatively be decomposed into a counterclockwise rotating vector of length 1.5 (red), and a clockwise rotating vector of length 0.5 (blue).



The Poincare Sphere

- Treat 2ψ and 2χ as longitude and latitude on sphere of radius $A=E^2$





Rohlfs & Wilson

Stokes parameters

- Spherical coordinates: radius I, axes Q, U, V
  - $I = E_X^2 + E_Y^2 = E_R^2 + E_L^2$
  - $Q = I \cos 2\chi \cos 2\psi = E_X^2 - E_Y^2 = 2\, E_R\, E_L \cos \delta_{RL}$
  - $U = I \cos 2\chi \sin 2\psi = 2\, E_X\, E_Y \cos \delta_{XY} = 2\, E_R\, E_L \sin \delta_{RL}$
  - $V = I \sin 2\chi = 2\, E_X\, E_Y \sin \delta_{XY} = E_R^2 - E_L^2$
- Only 3 independent parameters:
  - wave polarization confined to surface of Poincare sphere
  - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I,Q,U,V
  - defined by George Stokes (1852)
  - form complete description of wave polarization
  - NOTE: above true for 100% polarized monochromatic wave!

Linear Polarization

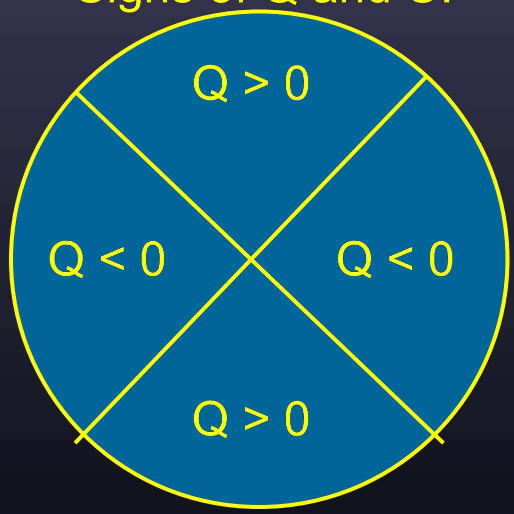
- Linearly Polarized Radiation: $V = 0$
  - Linearly polarized flux:

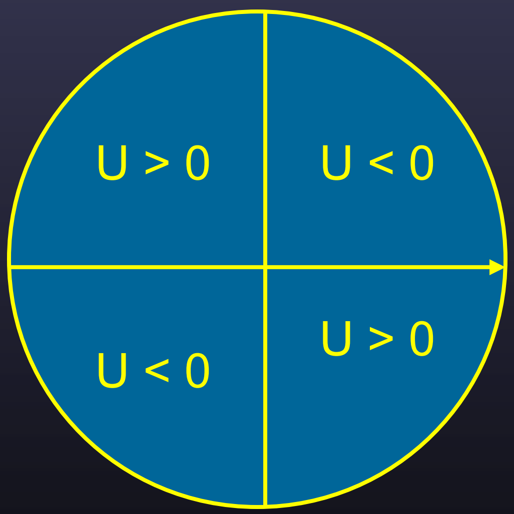
$$P = \sqrt{Q^2 + U^2}$$

  - Q and U define the linear polarization position angle:

$$\tan 2\psi = U / Q$$

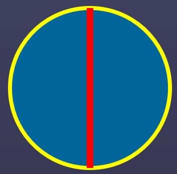
  - Signs of Q and U:



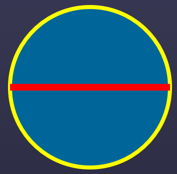


Simple Examples

- If V = 0, the wave is linearly polarized. Then,
  - If U = 0, and Q positive, then the wave is vertically polarized, Ψ=0°



  - If U = 0, and Q negative, the wave is horizontally polarized, Ψ=90°

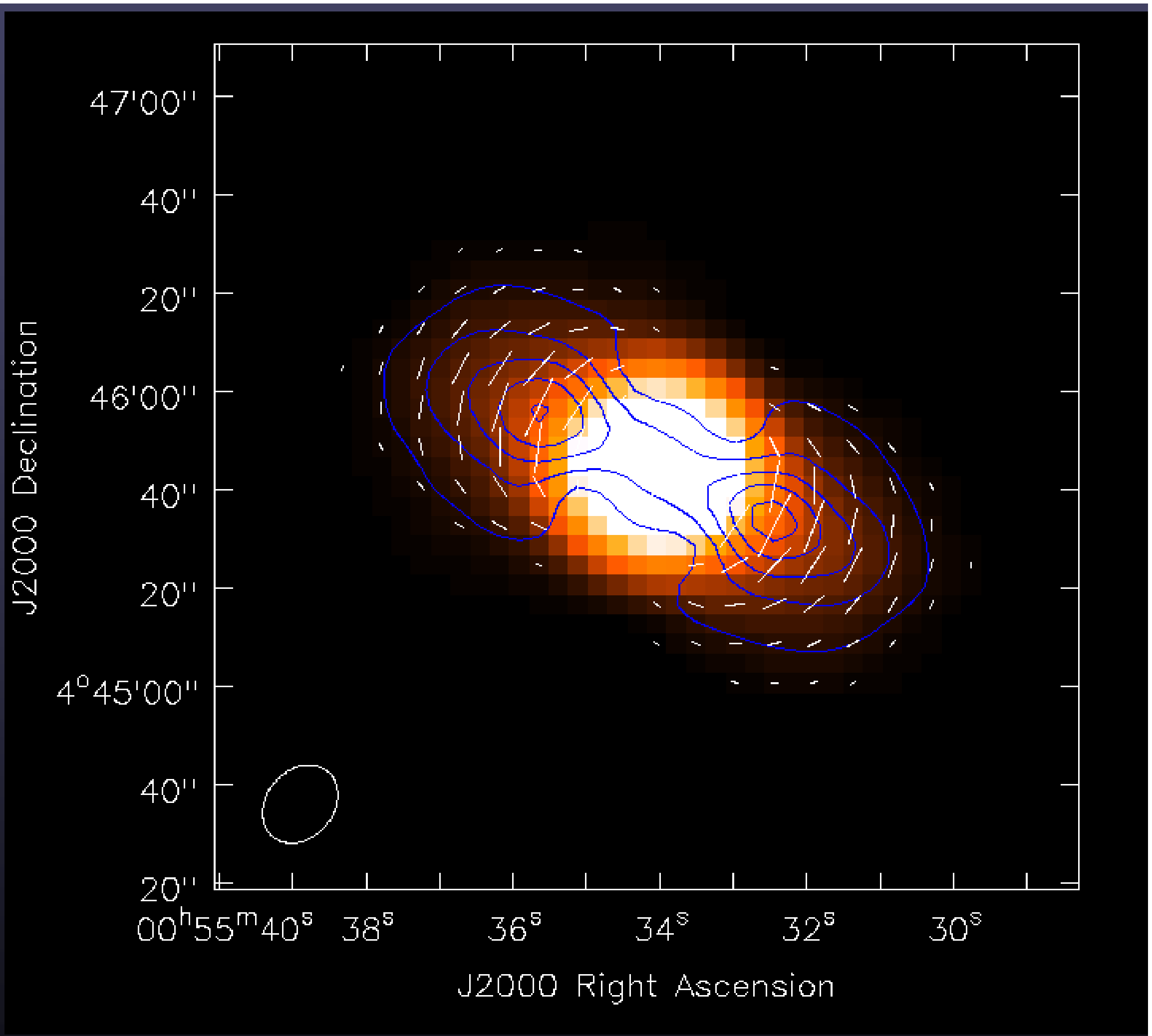


  - If Q = 0, and U positive, the wave is polarized at Ψ = 45°

  - If Q = 0, and U negative, the wave is polarized at Ψ = -45°.

Illustrative Example: Non-thermal Emission from Jupiter

- Apr 1999 VLA 5 GHz data
- D-config resolution is 14”
- Jupiter emits thermal radiation from atmosphere, plus polarized synchrotron radiation from particles in its magnetic field
- Shown is the I image (intensity) with polarization vectors rotated by 90° (to show B-vectors) and polarized intensity (blue contours)
- The polarization vectors trace Jupiter’s dipole
- Polarized intensity linked to the Io plasma torus



Why Use Stokes Parameters?

- Tradition
- They are scalar quantities, independent of basis XY, RL
- They have units of power (flux density when calibrated)
- They are simply related to actual antenna measurements.
- They easily accommodate the notion of partial polarization of non-monochromatic signals.
- We can (as I will show) make images of the I, Q, U, and V intensities directly from measurements made from an interferometer.
- These I,Q,U, and V images can then be combined to make images of the linear, circular, or elliptical characteristics of the radiation.

Non-Monochromatic Radiation, and Partial Polarization

- Monochromatic radiation is a myth.
- No such entity can exist (although it can be closely approximated).
- In real life, radiation has a finite bandwidth.
- Real astronomical emission processes arise from randomly placed, independently oscillating emitters (electrons).
- We observe the summed electric field, using instruments of finite bandwidth.
- Despite the chaos, polarization still exists, but is not complete – partial polarization is the rule.
- Stokes parameters defined in terms of mean quantities:

Stokes Parameters for Partial Polarization

$$I = \langle E_x^2 \rangle + \langle E_y^2 \rangle = \langle E_r^2 \rangle + \langle E_l^2 \rangle$$

$$Q = \langle E_x^2 \rangle - \langle E_y^2 \rangle = 2\langle E_r E_l \cos \delta_{rl} \rangle$$

$$U = 2\langle E_x E_y \cos \delta_{xy} \rangle = 2\langle E_r E_l \sin \delta_{rl} \rangle$$

$$V = 2\langle E_x E_y \sin \delta_{xy} \rangle = \langle E_r^2 \rangle - \langle E_l^2 \rangle$$

Note that now, unlike monochromatic radiation, the radiation is not necessarily 100% polarized.

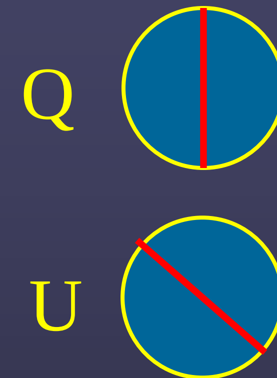
$$I^2 \geq Q^2 + U^2 + V^2$$

Summary – Fundamentals

- Monochromatic waves are polarized
- Expressible as 2 orthogonal independent transverse waves
  - elliptical cross-section ➔ polarization ellipse
  - 3 independent parameters
  - choice of basis, e.g. linear or circular
- Poincare sphere convenient representation
  - Stokes parameters I, Q, U, V
  - I intensity; Q,U linear polarization, V circular polarization
- Quasi-monochromatic “waves” in reality
  - can be partially polarized
  - still represented by Stokes parameters

Antenna Polarization

Measuring Polarization on the sky



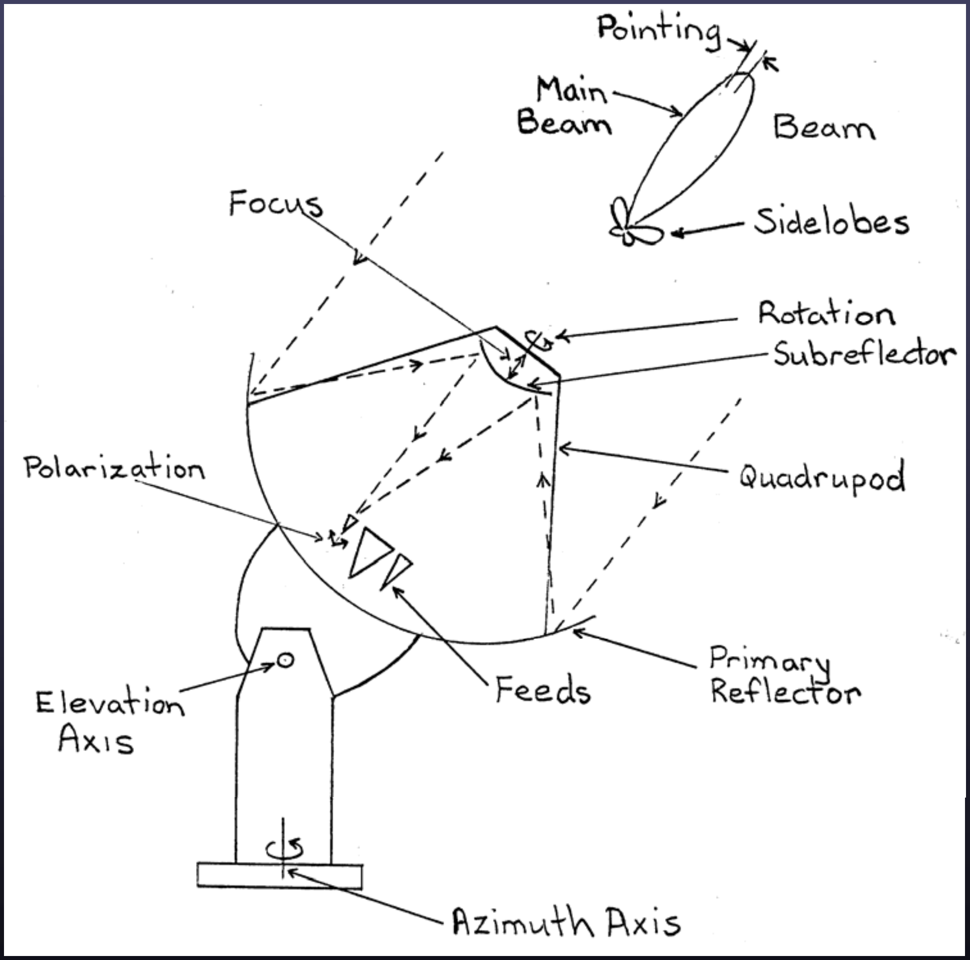
- Coordinate system dependence:
  - I independent
  - V depends on choice of “handedness”
    - V > 0 for RCP
  - Q,U depend on choice of “North” (plus handedness)
    - Q “points” North, U 45 toward East
- Polarization Angle Ψ

$$\Psi = \tfrac{1}{2} \tan^{-1} (U/Q) \quad \text{(North through East)}$$

  - also called the “electric vector position angle” (EVPA)
  - by convention, traces E-field vector (e.g. for synchrotron)
  - B-vector is perpendicular to this

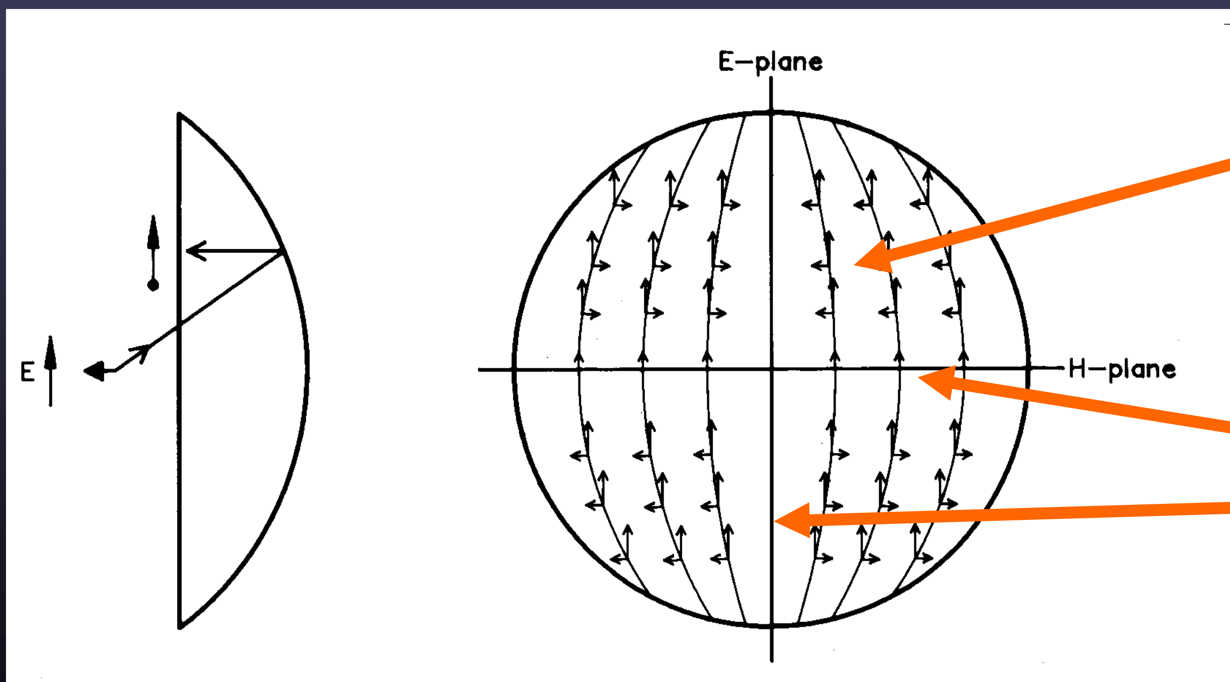
Optics – Cassegrain radio telescope

- Paraboloid illuminated by feedhorn:



Optics – telescope response

- Reflections
  - turn RCP ⇔ LCP
  - E-field (currents) allowed only in plane of surface
- "Field distribution" on aperture for E and B planes:

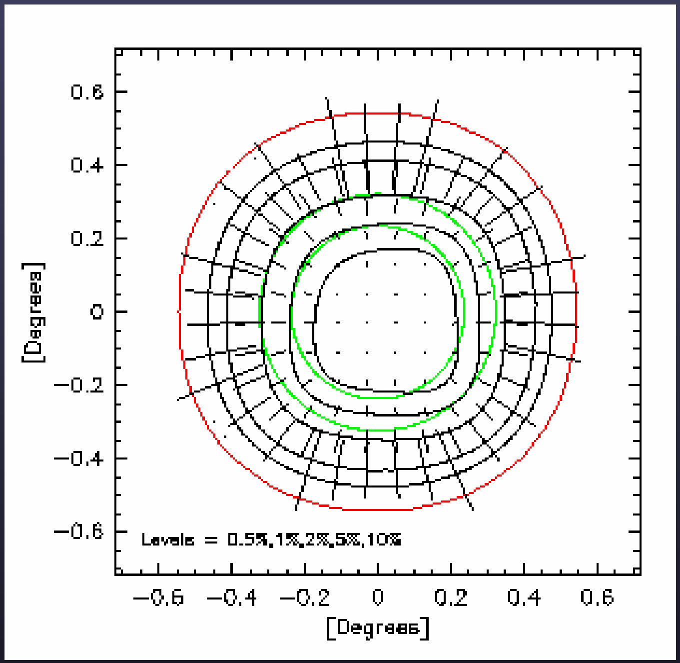


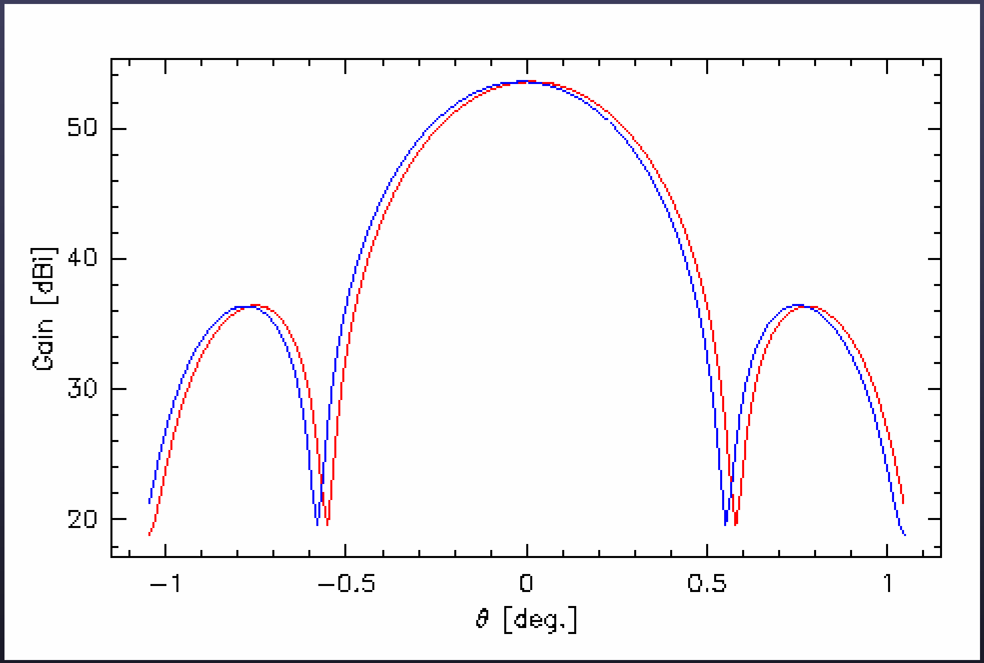
Cross-polarization at 45°

No cross-polarization on axes

Example – simulated VLA patterns

- EVLA Memo 58 “Using Grasp8 to Study the VLA Beam” W. Brisken



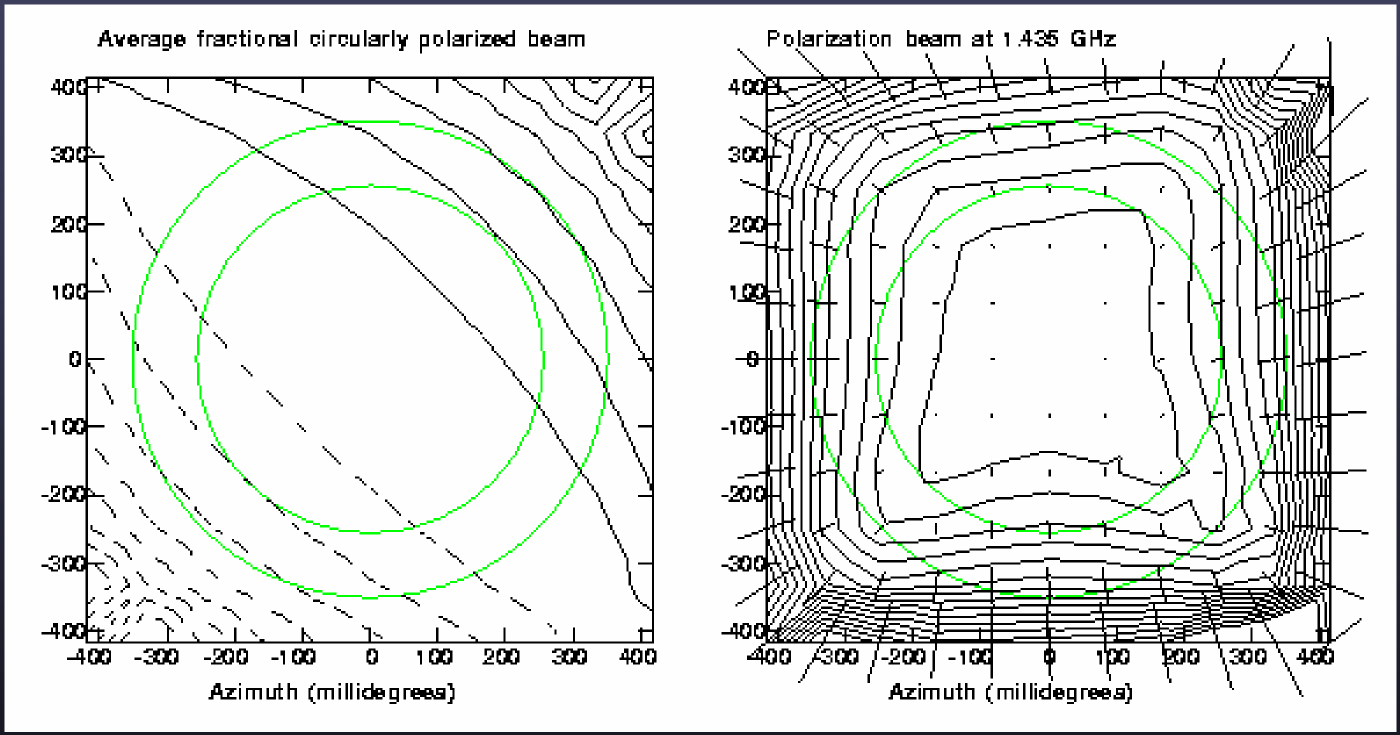


Linear Polarization

Circular Polarization cuts in R & L

Example – measured VLA patterns

- AIPS Memo 86 "Widefield Polarization Correction of VLA Snapshot Images at 1.4 GHz" W. Cotton (1994)



Circular Polarization

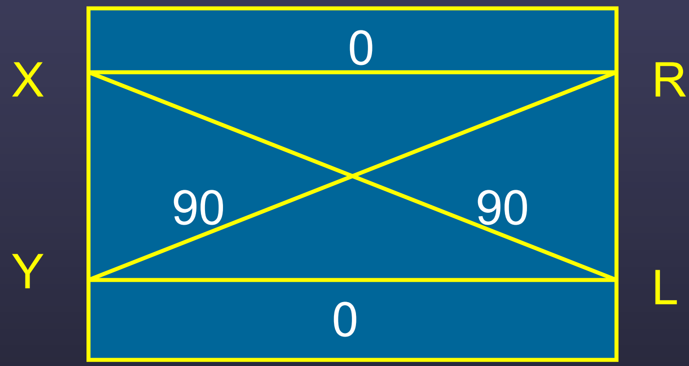
Linear Polarization

Polarization Reciever Outputs

- To do polarimetry (measure the polarization state of the EM wave), the antenna must have two outputs which respond differently to the incoming elliptically polarized wave.
- It would be most convenient if these two outputs are proportional to either:
  - The two linear orthogonal Cartesian components, (E_X, E_Y) as in ATCA and ALMA
  - The two circular orthogonal components, (E_R, E_L) as in VLA
- Sadly, this is not the case in general.
  - In general, each port is elliptically polarized, with its own polarization ellipse, with its p.a. and ellipticity.
- However, as long as these are different, polarimetry can be done.

Polarizers: Quadrature Hybrids

- We've discussed the two bases commonly used to describe polarization.
- It is quite easy to transform signals from one to the other, through a real device known as a 'quadrature hybrid'.



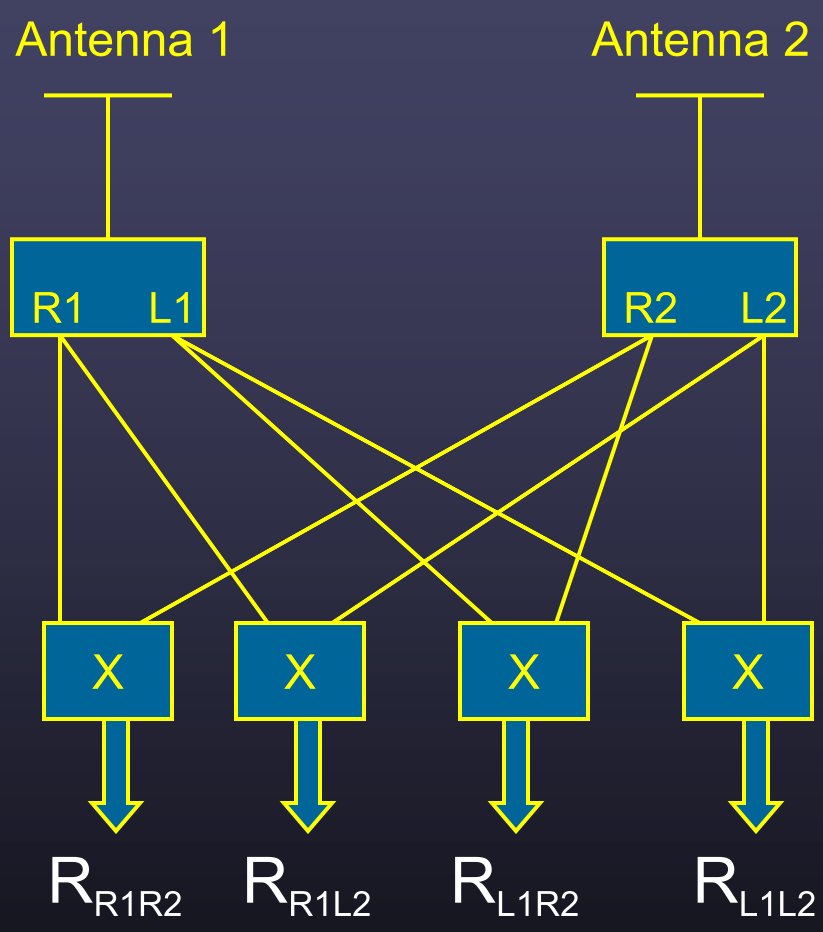
<u>Four Port Device:</u>
2 port input
2 ports output
mixing matrix

- To transform correctly, the phase shifts must be exactly 0 and 90 for all frequencies, and the amplitudes balanced.
- Real hybrids are imperfect – generate errors (mixing/leaking)
- Other polarizers (e.g. waveguide septum, grids) equivalent

Polarization Interferometry

Four Complex Correlations per Pair

- Two antennas, each with two differently polarized outputs, produce four complex correlations.
- From these four outputs, we want to make four Stokes Images.



Outputs: Polarization Vectors

- Each telescope receiver has two outputs
  - should be orthogonal, close to X,Y or R,L
  - even if single pol output, convenient to consider both possible polarizations (e.g. for leakage)
  - put into vector

$$\vec{E}(t) = \begin{pmatrix} E_R(t) \\ E_L(t) \end{pmatrix} \quad \text{or} \quad \vec{E}(t) = \begin{pmatrix} E_X(t) \\ E_Y(t) \end{pmatrix}$$

Correlation products: coherency vector

- Coherency vector: outer product of 2 antenna vectors as averaged by correlator

$$\vec{v}_{ij} = \left\langle \vec{E}_{\mathrm{i}} \otimes \vec{E}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} E^p \\ E^q \end{pmatrix}_i \otimes \begin{pmatrix} E^p \\ E^q \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \left\langle E_i^p \cdot E_j^{*p} \right\rangle \\ \left\langle E_i^p \cdot E_j^{*q} \right\rangle \\ \left\langle E_i^q \cdot E_j^{*p} \right\rangle \\ \left\langle E_i^q \cdot E_j^{*q} \right\rangle \end{pmatrix} = \begin{pmatrix} v^{pp} \\ v^{pq} \\ v^{qp} \\ v^{qq} \end{pmatrix}_{ij}$$

  - these are essentially the uncalibrated *visibilities* **v**
    - circular products RR, RL, LR, LL
    - linear products XX, XY, YX, YY
  - need to include corruptions before and after correlation

Polarization Products: General Case

$$
\begin{aligned}
v^{pq} = \tfrac{1}{2} G_{pq} \{ & I[\cos(\Psi_p - \Psi_q)\cos(\chi_p - \chi_q) + i\sin(\Psi_p - \Psi_q)\sin(\chi_p + \chi_q)] \\
& + Q[\cos(\Psi_p + \Psi_q)\cos(\chi_p + \chi_q) + i\sin(\Psi_p + \Psi_q)\sin(\chi_p - \chi_q)] \\
& - iU[\cos(\Psi_p + \Psi_q)\sin(\chi_p - \chi_q) + i\sin(\Psi_p + \Psi_q)\cos(\chi_p + \chi_q)] \\
& - V[\cos(\Psi_p - \Psi_q)\sin(\chi_p + \chi_q) + i\sin(\Psi_p - \Psi_q)\cos(\chi_p - \chi_q)]\}
\end{aligned}
$$

What are all these symbols?

v^{pq} is the complex output from the interferometer, for polarizations p and q from antennas 1 and 2, respectively.

Ψ and χ are the antenna polarization major axis and ellipticity for states p and q.

I,Q, U, and V are the Stokes Visibilities describing the polarization state of the astronomical signal.

G is the gain, which falls out in calibration.

WE WILL ABSORB FACTOR ½ INTO GAIN!!!!!!!

Coherency vector and Stokes vector

- Maps (perfect) visibilities to the Stokes vector **s**
- Example: circular polarization (e.g. VLA)

$$\vec{v}_{circ} = \mathbf{S}_{circ}\,\vec{s} = \begin{pmatrix} v^{RR} \\ v^{RL} \\ v^{LR} \\ v^{LL} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

- Example: linear polarization (e.g. ALMA, ATCA)

$$\vec{v}_{lin} = \mathbf{S}_{lin}\,\vec{s} = \begin{pmatrix} v^{XX} \\ v^{XY} \\ v^{YX} \\ v^{YY} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

Corruptions: Jones Matrices

- Antenna-based corruptions
  - pre-correlation polarization-dependent effects act as a matrix muliplication. This is the Jones matrix:

$$\vec{E}^{out} = \mathbf{J}\,\vec{E}^{in} \qquad \mathbf{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \qquad \vec{E} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

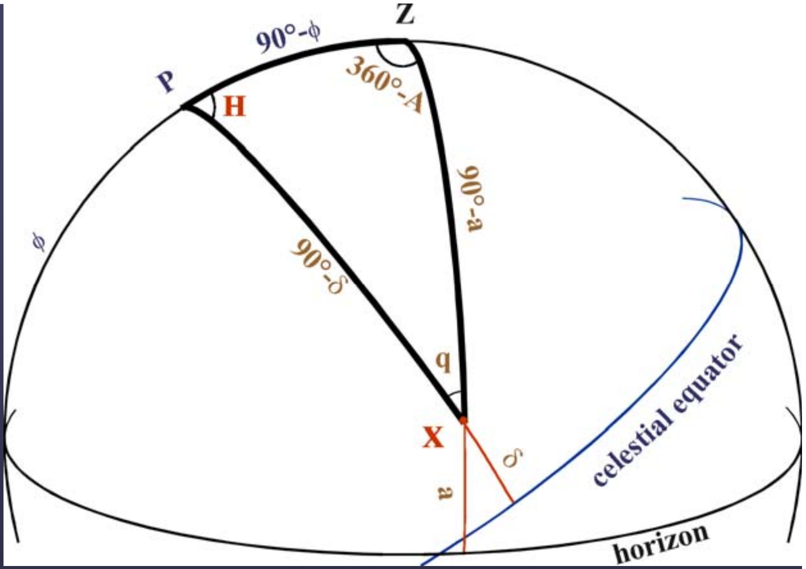
  - form of **J** depends on basis (RL or XY) and effect
    - off-diagonal terms J_{12} and J_{21} cause corruption (mixing)
  - total **J** is a string of Jones matrices for each effect

$$\mathbf{J} = \mathbf{J}_F\,\mathbf{J}_E\,\mathbf{J}_D\,\mathbf{J}_P$$

- Faraday, polarized beam, leakage, parallactic angle

Parallactic Angle, *P*

- Orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az telescopes
  - Rotates the position angle of linearly polarized radiation (R-L phase)



$$\mathbf{J}_P^{RL} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}; \quad \mathbf{J}_P^{XY} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

  - defined per antenna (often same over array)

$$\phi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))} \right)$$

$$l = \text{latitude}, \;\; h(t) = \text{hour angle}, \;\; \delta = \text{declination}$$

  - P modulation can be used to aid in calibration

Visibilities to Stokes on-sky: RL basis

- the (outer) products of the parallactic angle (P) and the Stokes matrices gives

$$\vec{v} = \mathbf{J}_P \, \mathbf{S} \, \vec{s}$$

- this matrix maps a sky Stokes vector to the coherence vector representing the four perfect (circular) polarization products:

$$\begin{pmatrix} v^{RR} \\ v^{RL} \\ v^{LR} \\ v^{LL} \end{pmatrix} = \begin{pmatrix} e^{-i(\phi_i-\phi_j)} & 0 & 0 & e^{-i(\phi_i-\phi_j)} \\ 0 & e^{-i(\phi_i+\phi_j)} & ie^{-i(\phi_i+\phi_j)} & 0 \\ 0 & e^{i(\phi_i+\phi_j)} & -ie^{i(\phi_i+\phi_j)} & 0 \\ e^{i(\phi_i-\phi_j)} & 0 & 0 & -e^{i(\phi_i-\phi_j)} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \xrightarrow[\phi_i=\phi_j=\phi]{} \begin{pmatrix} I+V \\ (Q+iU)e^{-i2\phi} \\ (Q-iU)e^{i2\phi} \\ I-V \end{pmatrix}$$

Circular Feeds: linear polarization in cross hands, circular in parallel-hands

Visibilities to Stokes on-sky: XY basis

- we have

$$\begin{pmatrix} v^{XX} \\ v^{XY} \\ v^{YX} \\ v^{YY} \end{pmatrix} = \begin{pmatrix} \cos(\phi_i - \phi_j) & \cos(\phi_i + \phi_j) & -\sin(\phi_i + \phi_j) & i\sin(\phi_i - \phi_j) \\ -\sin(\phi_i - \phi_j) & \sin(\phi_i + \phi_j) & \cos(\phi_i + \phi_j) & i\cos(\phi_i - \phi_j) \\ \sin(\phi_i - \phi_j) & \sin(\phi_i + \phi_j) & \cos(\phi_i + \phi_j) & -i\cos(\phi_i - \phi_j) \\ \cos(\phi_i - \phi_j) & -\cos(\phi_i + \phi_j) & -\sin(\phi_i + \phi_j) & i\sin(\phi_i - \phi_j) \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

- and for identical parallactic angles ϕ between antennas:

$$\begin{pmatrix} v^{XX} \\ v^{XY} \\ v^{YX} \\ v^{YY} \end{pmatrix} \xrightarrow[\phi_i = \phi_j = \phi]{} \begin{pmatrix} I + Q\cos 2\phi - U\sin 2\phi \\ Q\sin 2\phi + U\cos 2\phi + iV \\ Q\sin 2\phi + U\cos 2\phi - iV \\ I - Q\cos 2\phi + U\sin 2\phi \end{pmatrix}$$

Linear Feeds:
linear polarization in all hands, circular only in cross-hands

Basic Interferometry equations

- An interferometer naturally measures the transform of the sky intensity in uv-space convolved with aperture
  - cross-correlation of aperture voltage patterns in uv-plane
  - its tranform on sky is the primary beam **A** with FWHM ~ λ/D

$$V(\mathbf{u}) = \int d^2\mathbf{x}\, A(\mathbf{x}-\mathbf{x}_p)\, I(\mathbf{x})\, e^{-2\pi i\, \mathbf{u}\cdot(\mathbf{x}-\mathbf{x}_p)} + \mathrm{n}$$
$$= \int d^2\mathbf{v}\, \tilde{A}(\mathbf{u}-\mathbf{v})\, \tilde{I}(\mathbf{v})\, e^{2\pi i\, \mathbf{v}\cdot\mathbf{x}_p} + \mathrm{n}$$

  - The “tilde” quantities are Fourier transforms, with convention:

$$\tilde{T}(\mathbf{u}) = \int d^2\mathbf{x}\, e^{-i2\pi\mathbf{u}\cdot\mathbf{x}}\, T(\mathbf{x}) \qquad \mathbf{x} = (l,m) \leftrightarrow \mathbf{u} = (u,v)$$
$$T(\mathbf{x}) = \int d^2\mathbf{u}\, e^{i2\pi\mathbf{u}\cdot\mathbf{x}}\, \tilde{T}(\mathbf{u})$$

Polarization Interferometry : Q & U

- Parallel-hand & Cross-hand correlations (circular basis)
  - visibility *k* (antenna pair *ij* , time, pointing *x*, channel ν, noise n):

$$V_k^{RR}(\mathbf{u}_k) = \int d^2\mathbf{v}\,\tilde{A}_k^{RR}(\mathbf{u}_k - \mathbf{v})[\tilde{I}_\nu(\mathbf{v}) + \tilde{V}_\nu(\mathbf{v})]e^{2\pi i\,\mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{RR}$$

$$V_k^{RL}(\mathbf{u}_k) = \int d^2\mathbf{v}\,\tilde{A}_k^{RL}(\mathbf{u}_k - \mathbf{v})[\tilde{Q}_\nu(\mathbf{v}) + i\tilde{U}_\nu(\mathbf{v})]e^{-i2\phi_k}\,e^{2\pi i\,\mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{RL}$$

$$V_k^{LR}(\mathbf{u}_k) = \int d^2\mathbf{v}\,\tilde{A}_k^{LR}(\mathbf{u}_k - \mathbf{v})[\tilde{Q}_\nu(\mathbf{v}) - i\tilde{U}_\nu(\mathbf{v})]e^{i2\phi_k}\,e^{2\pi i\,\mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{LR}$$

$$V_k^{LL}(\mathbf{u}_k) = \int d^2\mathbf{v}\,\tilde{A}_k^{LL}(\mathbf{u}_k - \mathbf{v})[\tilde{I}_\nu(\mathbf{v}) - \tilde{V}_\nu(\mathbf{v})]e^{2\pi i\,\mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{LL}$$

  - where kernel A is the aperture cross-correlation function, ϕ is the parallactic angle, and **Q+iU=P** is the complex linear polarization

$$\tilde{P}(\mathbf{v}) = \tilde{Q}(\mathbf{v}) + i\tilde{U}(\mathbf{v}) = \left|\tilde{P}(\mathbf{v})\right| e^{i2\varphi(\mathbf{v})}$$

    - the phase of **P** is φ (the R-L phase difference)

Example: RL basis imaging

- Parenthetical Note:
  - can make a pseudo-I image by gridding RR+LL on the Fourier half-plane and inverting to a real image
  - can make a pseudo-V image by gridding RR-LL on the Fourier half-plane and inverting to real image
  - can make a pseudo-(Q+iU) image by gridding RL to the full Fourier plane (with LR as the conjugate) and inverting to a complex image
  - does not require having full polarization RR,RL,LR,LL for every visibility
- More on imaging (& deconvolution) tomorrow!

Polarization Leakage, *D*

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed systems have *d* < 1-5% (but some systems >10% ☹)
  - A geometric property of the antenna, feed & polarizer design
    - frequency dependent (e.g. quarter-wave at center ν)
    - direction dependent (in beam) due to antenna
  - For *R,L* systems
    - parallel hands affected as d•Q + d•U , so only important at high dynamic range (because *Q,U~d*, typically)
    - cross-hands affected as d•I so almost always important

$$\mathbf{J}_D^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Leakage of q into p (e.g. L into R)

Leakage revisited…

- Primary on-axis effect is "leakage" of one polarization into the measurement of the other (e.g. R ⇔ L)
  - but, direction dependence due to polarization beam!
- Customary to factor out on-axis leakage into D and put direction dependence in "beam"
  - example: expand RL basis with on-axis leakage

$$\hat{V}_{ij}^{RR} = V_{ij}^{RR} + d_i^R V_{ij}^{LR} + d_j^{*R} V_{ij}^{RL} + d_i^R d_j^{*R} V_{ij}^{LL}$$

$$\hat{V}_{ij}^{RL} = V_{ij}^{RL} + d_i^R V_{ij}^{LL} + d_j^{*L} V_{ij}^{RR} + d_i^R d_j^L V_{ij}^{LR}$$

  - similarly for XY basis

Example: RL basis leakage

- In full detail:

"true" signal

2nd order: D•P into I

2nd order: D²•I into I

1st order: D•I into P

3rd order: D²•P* into P

$$V_{ij}^{RR} = \int_{sky} E_{ij}^{RR}(l,m)[(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$
$$+d_i^R e^{-i(\chi_i+\chi_j)}(\mathrm{Q}-i\mathrm{U}) + d_j^{*R} e^{i(\chi_i+\chi_j)}(\mathrm{Q}+i\mathrm{U})$$
$$+d_i^R d_j^{*R} e^{-i(\chi_i-\chi_j)}(\mathrm{I}-\mathrm{V})](l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

$$V_{ij}^{RL} = \int_{sky} E_{ij}^{RL}(l,m)[(\mathrm{Q}+i\mathrm{U})e^{i(\chi_i+\chi_j)}$$
$$+d_i^R(\mathrm{I}-\mathrm{V})e^{-i(\chi_i-\chi_j)} + d_j^{*L}(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$
$$+d_i^R d_j^{*L}(\mathrm{Q}-i\mathrm{U})e^{-i(\chi_i+\chi_j)}](l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

Example: linearized leakage

- RL basis, keeping only terms linear in I,Q±iU,d:

$$V_{ij}^{RL} = (\mathrm{Q} + i\mathrm{U})e^{-i(\phi_i+\phi_j)} + \mathrm{I}\,(d_i^R\, e^{i(\phi_i-\phi_j)} + d_j^{*L}\, e^{-i(\phi_i-\phi_j)})$$
$$V_{ij}^{LR} = (\mathrm{Q} - i\mathrm{U})e^{i(\phi_i+\phi_j)} - \mathrm{I}\,(d_i^L\, e^{-i(\phi_i-\phi_j)} + d_j^{*R}\, e^{i(\phi_i-\phi_j)})$$

- Likewise for XY basis, keeping linear in I,Q,U,V,d,sin(ϕ_i-ϕ_j)

$$V_{ij}^{XY} = \mathrm{Q}\sin(\phi_{\mathrm{i}} + \phi_j) + \mathrm{U}\cos(\phi_i + \phi_j) + i\,\mathrm{V} + [(d_i^X + d_j^{*Y})\cos(\phi_i - \phi_j) - \sin(\phi_i - \phi_j)]\mathrm{I}$$
$$V_{ij}^{YX} = \mathrm{Q}\sin(\phi_{\mathrm{i}} + \phi_j) + \mathrm{U}\cos(\phi_i + \phi_j) + i\,\mathrm{V} + [(d_i^Y + d_j^{*X})\cos(\phi_i - \phi_j) + \sin(\phi_i - \phi_j)]\mathrm{I}$$

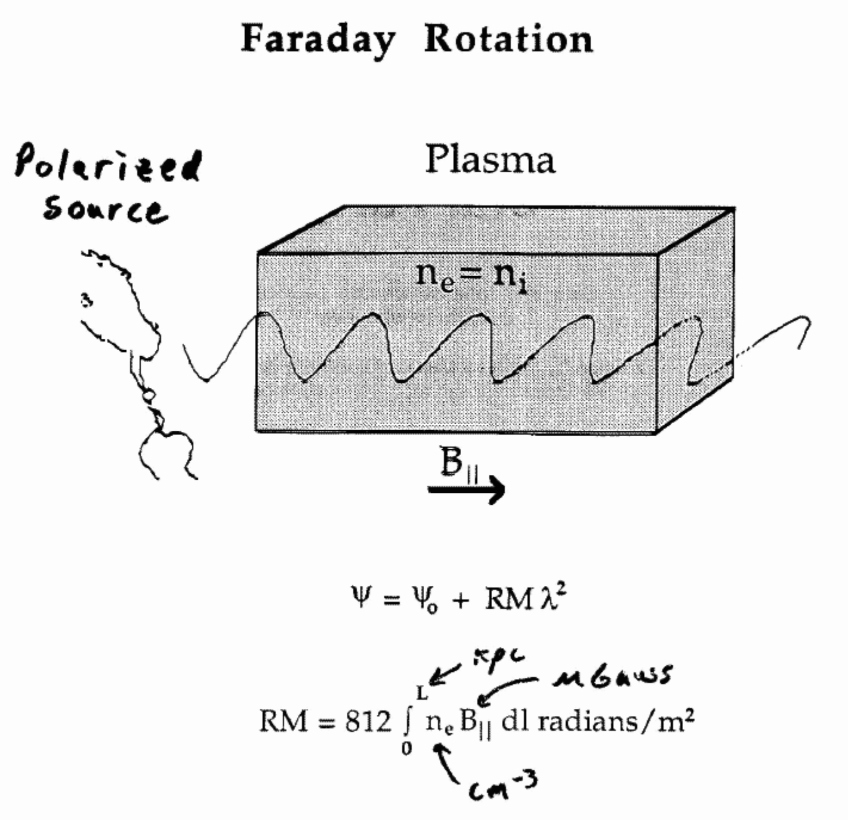
WARNING: Using linear order will limit dynamic range!

Ionospheric Faraday Rotation, *F*

- Birefringency due to magnetic field in ionospheric plasma

$$\mathbf{J}_F^{RL} = \begin{pmatrix} e^{i\Delta\phi} & 0 \\ 0 & e^{-i\Delta\phi} \end{pmatrix}$$

$$\mathbf{J}_F^{XY} = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi & \cos\Delta\phi \end{pmatrix}$$



is direction-dependent

$$\Delta\phi \approx 0.15^\circ \;\; \lambda^2 \int B_{||} n_e ds$$

$$(\lambda \text{ in cm},\; n_e ds \text{ in } 10^{14}\,\text{cm}^{-2},\; B_{||} \text{ in G})$$

$$TEC = \int n_e ds \;\; \sim 10^{14}\,\text{cm}^{-2}; \quad B_{||} \sim 1\text{G};$$

$$\lambda = 20\text{cm} \rightarrow \Delta\phi \sim 60^\circ$$

- also present in ISM, IGM and intrinsic to radio sources!
  - can come from different Faraday depths → tomography

Antenna voltage pattern, E

- Direction-dependent gain and polarization
  - includes primary beam
    - Fourier transform of cross-correlation of antenna voltage patterns
    - includes polarization asymmetry (squint)

$$\mathbf{J}_E^{pq} = \begin{pmatrix} e^{pp}(l',m') & e^{pq}(l',m') \\ e^{qp}(l',m') & e^{qq}(l',m') \end{pmatrix}$$

  - includes off-axis cross-polarization (leakage)
    - convenient to reserve D for on-axis leakage
  - important in wide-field imaging and mosaicing
    - when sources fill the beam (e.g. low frequency)

Summary – polarization interferometry

- Choice of basis: CP or LP feeds
  - usually a technology consideration
- Follow the signal path
  - ionospheric Faraday rotation F at low frequency
    - direction dependent (and antenna dependent for long baselines)
  - parallactic angle P for coordinate transformation to Stokes
    - antennas can have differing PA (e.g. VLBI)
  - “leakage” D varies with ν and over beam (mix with E)
- Leakage
  - use full (all orders) D solver when possible
  - linear approximation OK for low dynamic range
  - beware when antennas have different parallactic angles

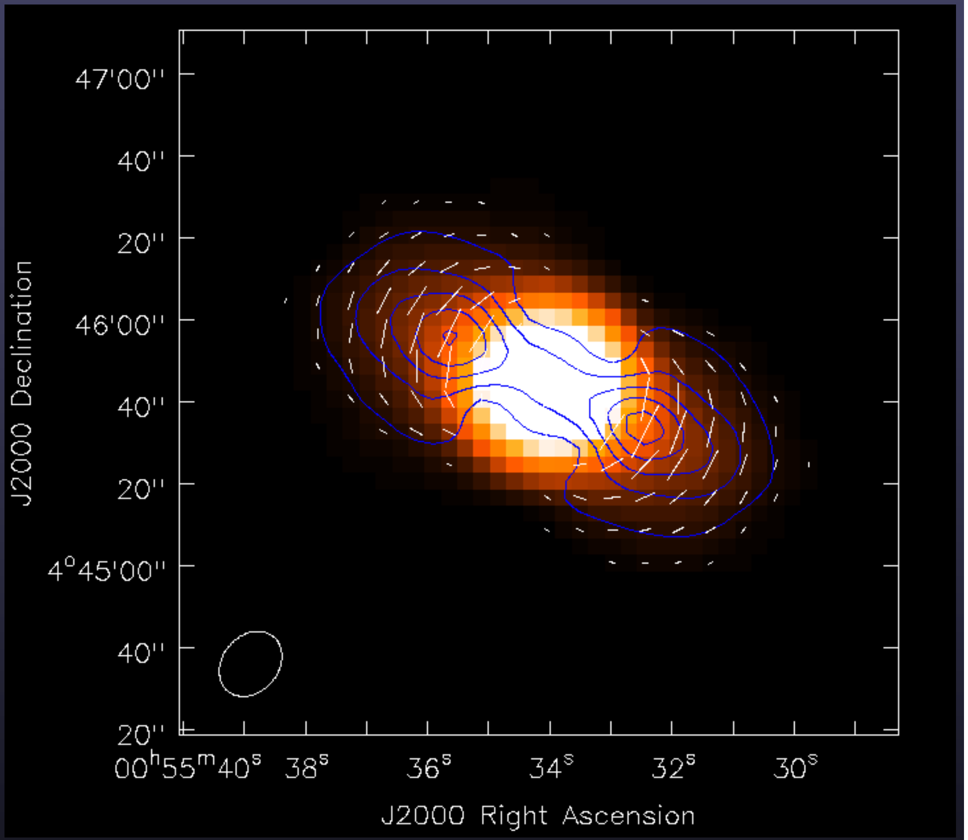
Polarization Calibration & Observation

So you want to make a polarization image…

- Making polarization images
  - follow general rules for imaging
  - image & deconvolve I, Q, U, V planes
  - Q, U, V will be positive and negative
  - V image can often be used as check
- Polarization vector plots
  - EVPA calibrator to set angle (e.g. R-L phase difference)

    Φ = ½ tan-1 U/Q for E vectors
  - B vectors ⊥ E
  - plot E vectors (length given by P)
- Leakage calibration is essential
- See Tutorials on Friday

e.g Jupiter 6cm continuum



Strategies for leakage calibration

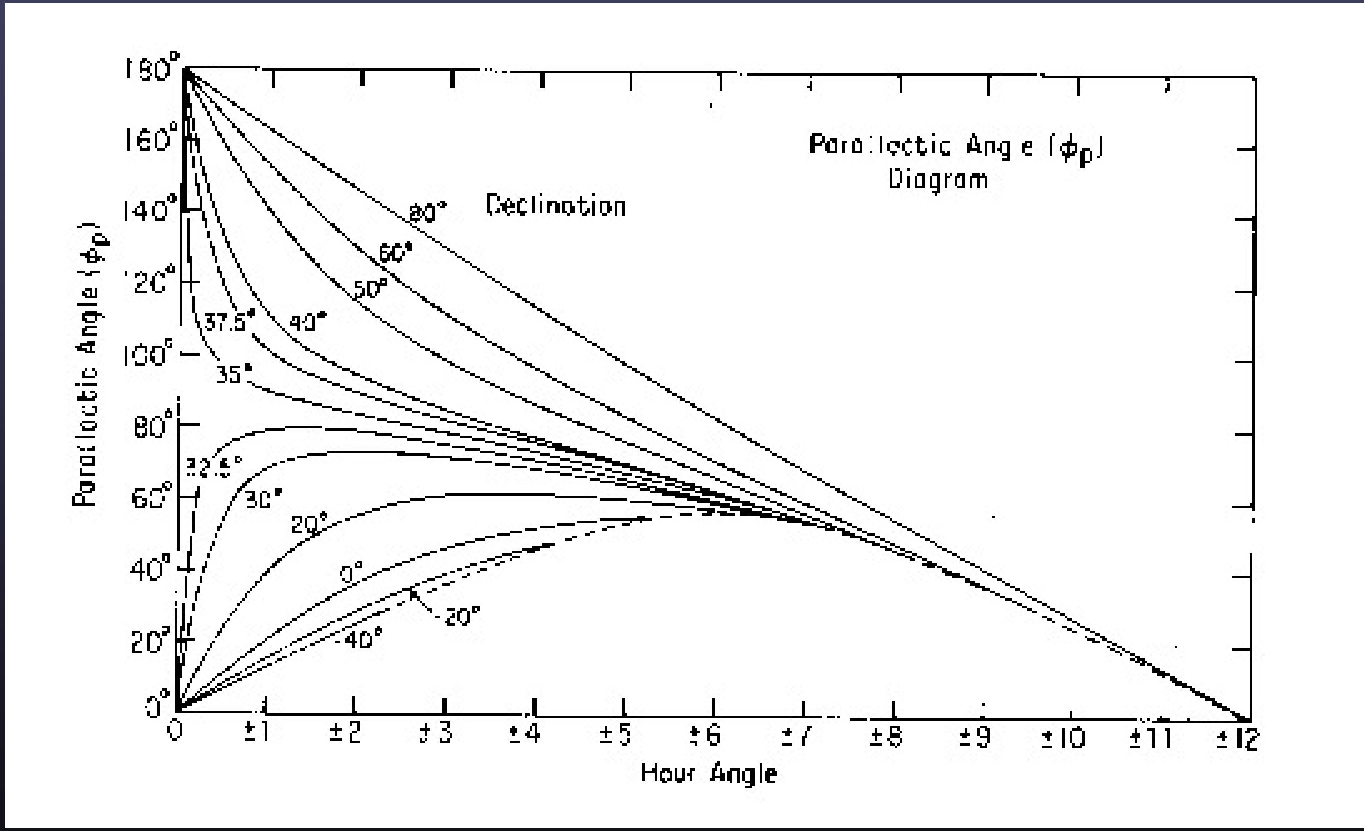
- Need a bright calibrator! Effects are low level…
  - determine antenna gains independently (mostly from parallel hands)
  - use cross-hands (mostly) to determine leakage
  - do matrix solution to go beyond linear order
- Calibrator is unpolarized
  - leakage directly determined (ratio to `I` model), but only to an overall complex constant (additive over array)
  - need way to fix phase δ_p-δ_q (*ie.* R-L phase difference), e.g. using another calibrator with known EVPA
- Calibrator of known (non-zero) linear polarization
  - leakage can be directly determined (for I,Q,U,V model)
  - unknown *p-q* phase can be determined (from U/Q etc.)

Other strategies

- Calibrator of unknown polarization
  - solve for model IQUV and D simultaneously or iteratively
  - need good parallactic angle coverage to modulate sky and instrumental signals
    - in instrument basis, sky signal modulated by $e^{i2\chi}$
- With a very bright strongly polarized calibrator
  - can solve for leakages and polarization per baseline
  - can solve for leakages using parallel hands!
- With no calibrator
  - hope it averages down over parallactic angle
  - transfer D from a similar observation
    - usually possible for several days, better than nothing!
    - need observations at same frequency

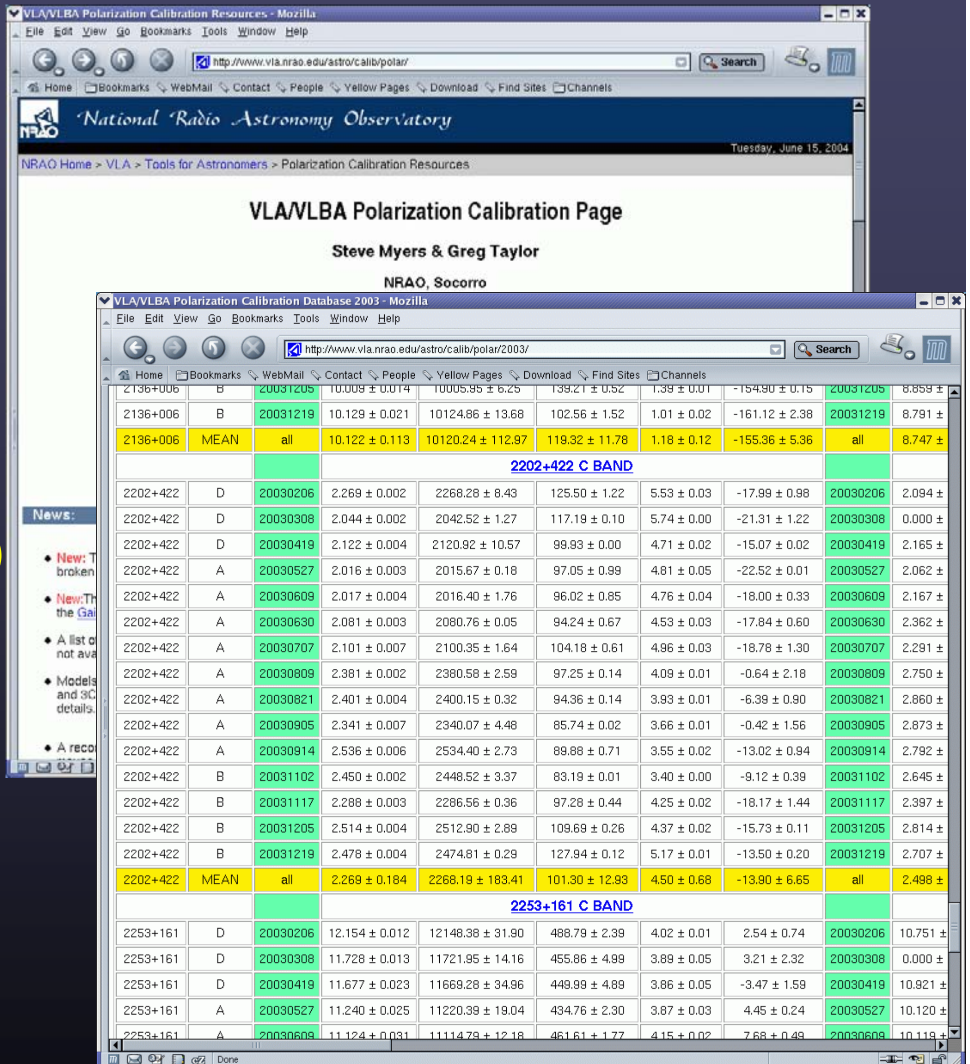
Parallactic Angle Coverage at VLA

- fastest PA swing for source passing through zenith
  - to get good PA coverage in a few hours, need calibrators between declination +20° and +60°



Finding polarization calibrators

- Standard sources
  - planets (unpolarized if unresolved)
  - 3C286, 3C48, 3C147 (known IQU, stable)
  - sources monitored (e.g. by VLA)
  - other bright sources (bootstrap)

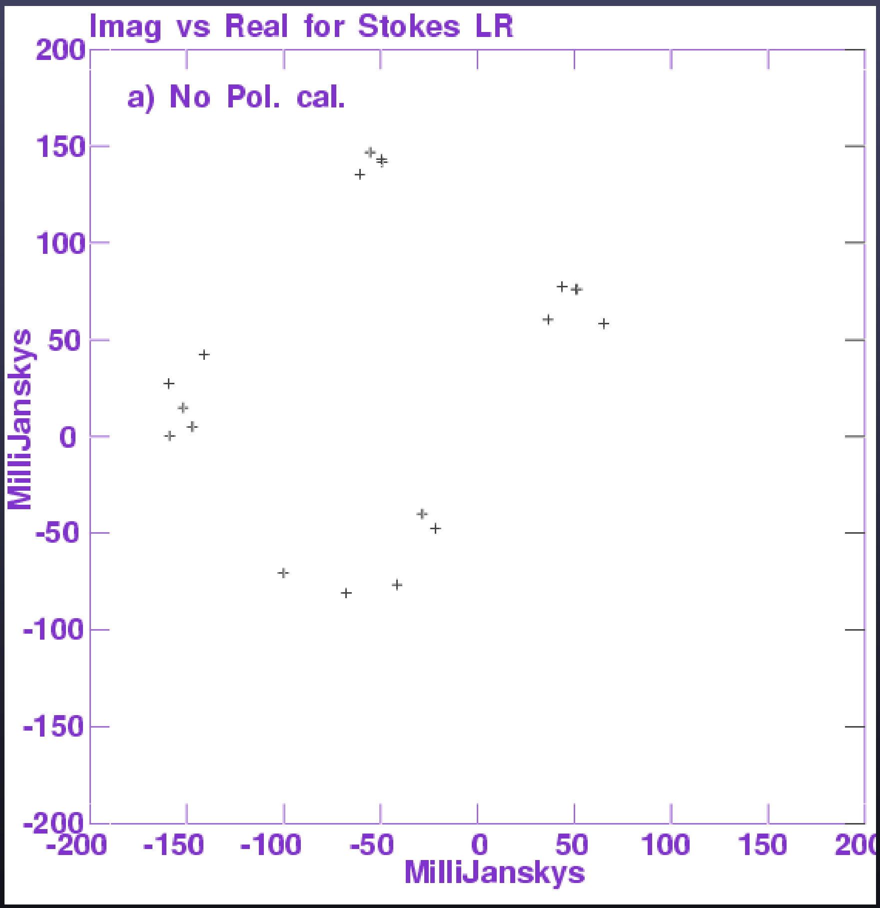


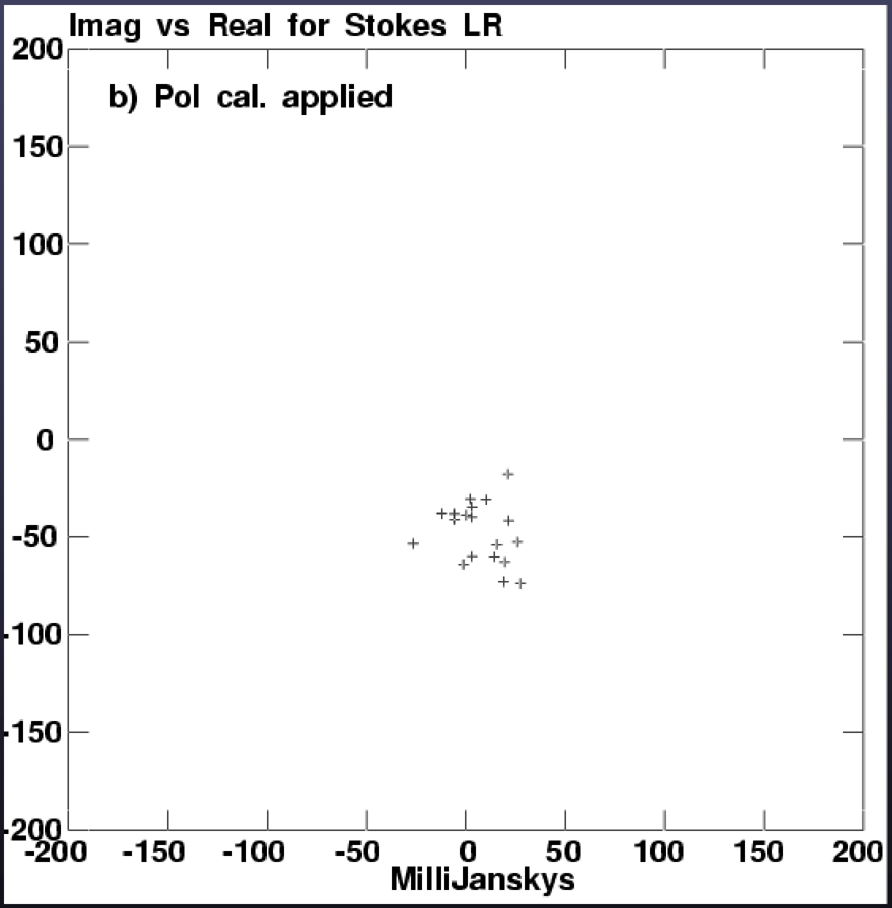
2136+006	B	20031205	10.009 ± 0.014	10005.95 ± 6.25	139.21 ± 0.52	1.39 ± 0.01	-154.90 ± 0.15	20031205	8.859 ±
2136+006	B	20031219	10.129 ± 0.021	10124.86 ± 13.68	102.56 ± 1.52	1.01 ± 0.02	-161.12 ± 2.38	20031219	8.791 ±
2136+006	MEAN	all	10.122 ± 0.113	10120.24 ± 112.97	119.32 ± 11.78	1.18 ± 0.12	-155.36 ± 5.36	all	8.747 ±
			2202+422 C BAND						
2202+422	D	20030206	2.269 ± 0.002	2268.28 ± 8.43	125.50 ± 1.22	5.53 ± 0.03	-17.99 ± 0.98	20030206	2.094 ±
2202+422	D	20030308	2.044 ± 0.002	2042.52 ± 1.27	117.19 ± 0.10	5.74 ± 0.00	-21.31 ± 1.22	20030308	0.000 ±
2202+422	D	20030419	2.122 ± 0.004	2120.92 ± 10.57	99.93 ± 0.00	4.71 ± 0.02	-15.07 ± 0.02	20030419	2.165 ±
2202+422	A	20030527	2.016 ± 0.003	2015.67 ± 0.18	97.05 ± 0.99	4.81 ± 0.05	-22.52 ± 0.01	20030527	2.062 ±
2202+422	A	20030609	2.017 ± 0.004	2016.40 ± 1.76	96.02 ± 0.85	4.76 ± 0.04	-18.00 ± 0.33	20030609	2.167 ±
2202+422	A	20030630	2.081 ± 0.003	2080.76 ± 0.05	94.24 ± 0.67	4.53 ± 0.03	-17.84 ± 0.60	20030630	2.362 ±
2202+422	A	20030707	2.101 ± 0.007	2100.35 ± 1.64	104.18 ± 0.61	4.96 ± 0.03	-18.78 ± 1.30	20030707	2.291 ±
2202+422	A	20030809	2.381 ± 0.002	2380.58 ± 2.59	97.25 ± 0.14	4.09 ± 0.01	-0.64 ± 2.18	20030809	2.750 ±
2202+422	A	20030821	2.401 ± 0.004	2400.15 ± 0.32	94.36 ± 0.14	3.93 ± 0.01	-6.39 ± 0.90	20030821	2.860 ±
2202+422	A	20030905	2.341 ± 0.007	2340.07 ± 4.48	85.74 ± 0.02	3.66 ± 0.01	-0.42 ± 1.56	20030905	2.873 ±
2202+422	A	20030914	2.536 ± 0.006	2534.40 ± 2.73	89.88 ± 0.71	3.55 ± 0.02	-13.02 ± 0.94	20030914	2.792 ±
2202+422	B	20031102	2.450 ± 0.002	2448.52 ± 3.37	83.19 ± 0.01	3.40 ± 0.00	-9.12 ± 0.39	20031102	2.645 ±
2202+422	B	20031117	2.288 ± 0.003	2286.56 ± 0.36	97.28 ± 0.44	4.25 ± 0.02	-18.17 ± 1.44	20031117	2.397 ±
2202+422	B	20031205	2.514 ± 0.004	2512.90 ± 2.89	109.69 ± 0.26	4.37 ± 0.02	-15.73 ± 0.11	20031205	2.814 ±
2202+422	B	20031219	2.478 ± 0.004	2474.81 ± 0.29	127.94 ± 0.12	5.17 ± 0.01	-13.50 ± 0.20	20031219	2.707 ±
2202+422	MEAN	all	2.269 ± 0.184	2268.19 ± 183.41	101.30 ± 12.93	4.50 ± 0.68	-13.90 ± 6.65	all	2.498 ±
			2253+161 C BAND						
2253+161	D	20030206	12.154 ± 0.012	12148.38 ± 31.90	488.79 ± 2.39	4.02 ± 0.01	2.54 ± 0.74	20030206	10.751 ±
2253+161	D	20030308	11.728 ± 0.013	11721.95 ± 14.16	455.86 ± 4.99	3.89 ± 0.05	3.21 ± 2.32	20030308	0.000 ±
2253+161	D	20030419	11.677 ± 0.023	11669.28 ± 34.96	449.99 ± 4.89	3.86 ± 0.05	-3.47 ± 1.59	20030419	10.921 ±
2253+161	A	20030527	11.240 ± 0.025	11220.39 ± 19.04	434.76 ± 2.30	3.87 ± 0.03	4.45 ± 0.24	20030527	10.120 ±
2253+161	A	20030609	11.124 ± 0.031	11114.79 ± 12.18	461.61 ± 1.77	4.15 ± 0.02	7.68 ± 0.49	20030609	10.119 ±

http://www.vla.nrao.edu/astro/calib/polar/

Example: D-term calibration

- D-term calibration effect on RL visibilities (should be Q+iU):

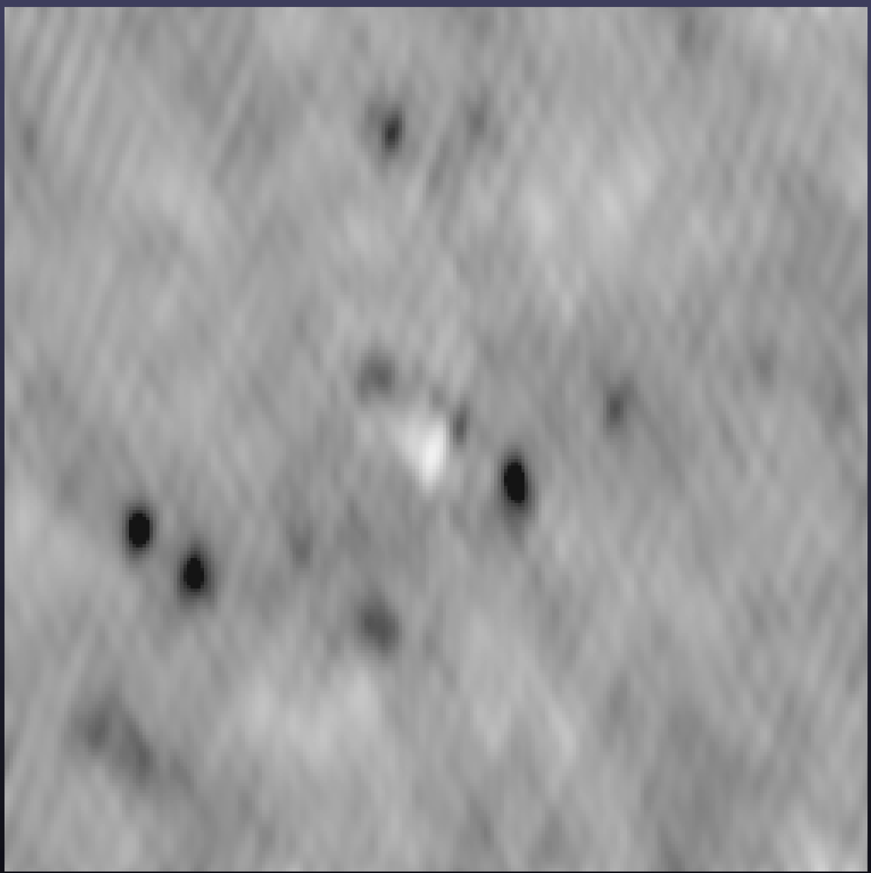




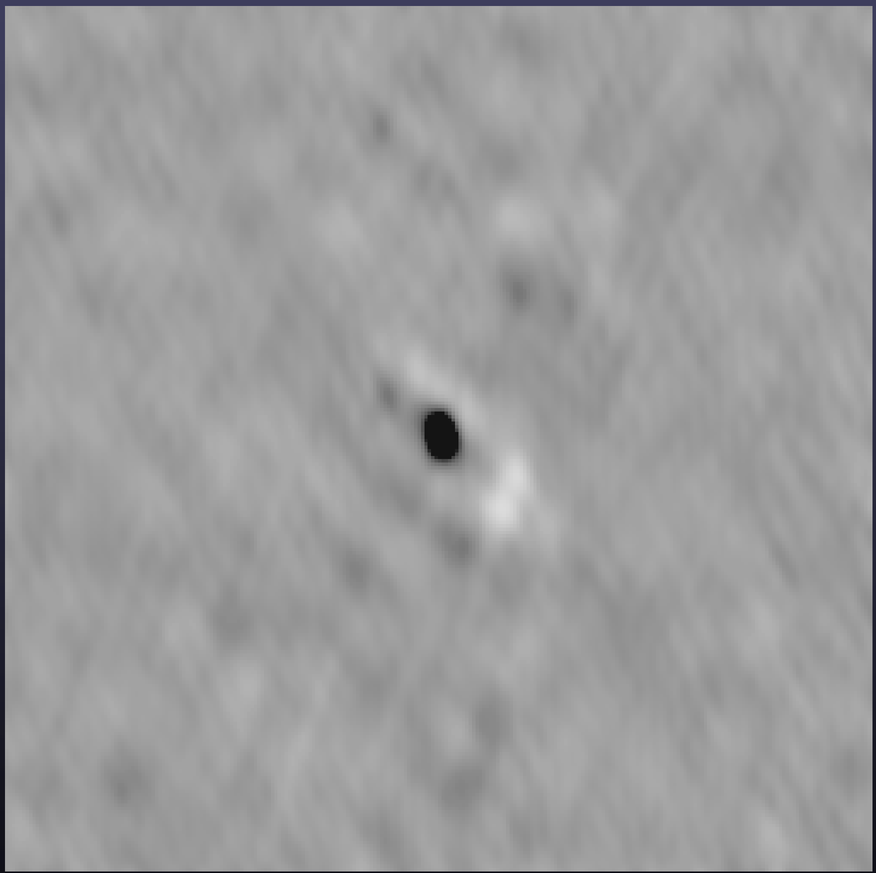
Example: D-term calibration

- D-term calibration effect in image plane :

Bad D-term solution



Good D-term solution



Summary – Observing & Calibration

- Follow normal calibration procedure (previous lecture)
- Need bright calibrator for leakage D calibration
  - best calibrator has strong known polarization
  - unpolarized sources also useful
- Parallactic angle coverage useful
  - necessary for unknown calibrator polarization
- Need to determine unknown *p-q* phase
  - CP feeds need EVPA calibrator for R-L phase
  - if system stable, can transfer from other observations
- Special Issues
  - observing CP difficult with CP feeds
  - wide-field polarization imaging (needed for EVLA & ALMA)