Electromagnetic Forces and Fields in a Rotating Reference Frame

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ABSTRACT

Maxwell's equations and the equations governing charged particle dynamics are presented for a rotating coordinate system with the global time coordinate of an observer on the rotational axis. Special care is taken in defining the relevant entities in these equations. Ambiguities in the definitions of the electromagnetic fields are pointed out, and in fact are shown to be essential in such a system of coordinates. The Lorentz force is found to have an extra term in this frame, which has its origins in relativistic mass. A related term in the energy equation, which allows inertia to be gained even during strict corotation, suggests ways existing pulsar magnetosphere models may be modified to match observed 'braking indices' more closely.

Subject headings: magnetic fields — relativity — pulsars: general

1. Introduction

In pulsar magnetosphere theory and other applications (Chedia *et al.* 1996; Cheng 1984; Fawley, Arons, & Scharlemann 1977; Hones & Bergeson 1965; Schiff 1939), it is often convenient to adopt a rotating coordinate system as a frame of reference. Such a frame is non-inertial, and has the usual Coriolis and centrifugal fictitious forces present as a well-known consequence. However, electrodynamic processes are often of primary interest, and this leads to several difficulties. In particular, the interpretation of various effects and even the definitions of the electric and magnetic fields are necessarily ambiguous (some consequences of this were noticed by Backus 1956). It is the purpose of this work to explore these matters in some detail and thereby provide clarification. In particular, we will first present Maxwell's equations and then the Lorentz force in a system of rotating Cartesian spatial coordinates, with the global time coordinate of a (stationary) observer on the axis of rotation.

As will be seen, it is difficult to define the electric and magnetic fields in this frame. This situation worsens with distance from the axis of rotation, and becomes critical at the 'light cylinder' distance, $r_L = 1/\Omega$, where Ω is the angular frequency of rotation (here and throughout, we use units where $c = 1$). This situation will be shown to be an essential feature of all such frames whose metric tensor $g_{\mu\nu}$ has off-diagonal elements. For this reason, all electromagnetic quantities used here will be given careful definitions, with reference to their values in an inertial non-rotating frame (where the ambiguities disappear).

Among the most surprising consequences which will be shown are that the Lorentz force acquires an extra 'relativistic mass' term in this frame, and particles may gain inertia even when undergoing strict corotation. Although negligible at low altitudes, these effects also can become critical at altitudes approaching the light cylinder. In pulsar models, it is precisely in this regime where the magnetic polar magnetospheric currents are expected to close; the details of this process remain an open question whose answer is of key importance for the models. It is therefore hoped that this paper will serve both as a reference for those who wish to adopt rotating coordinates when studying electrodynamic processes, and as a guide for interpreting the various phenomena in these coordinates.

2. Definitions

We must first define our coordinate systems; the remainder of the mathematics is then straightforward tensor algebra. To avoid an overabundance of 'primes,' all coordinate-dependent quantities with 'primes' attached shall refer to the inertial (non-rotating) frame, and 'unprimed' quantities will be used for the rotating frame. Greek letters will be used for indices which range over all four spacetime components, for example, $U^\alpha = (U^t, U^x, U^y, U^z)$, while Latin letters will be used when variation is to be made over the three ordinary spatial indices (x, y, z). The usual summation convention is adopted.

We start with an inertial nonrotating Cartesian coordinate system (t', x', y', z') with the 'flat' (ignoring gravity) Minkowski metric

$$(ds')^2 = -(dt')^2 + (dx')^2 + (dy')^2 + (dz')^2 \tag{1}$$

where ds' is the infinitesimal proper distance for a spacelike interval (and is coordinate-independent, although the 'prime' is left on for clarity). If the interval is instead timelike, we define its proper time $d\tau'$ (also coordinate-independent) by replacing $(ds')^2$ by $-(d\tau')^2$ above. We write the above as

$$(ds')^2 = g_{\mu'\nu'} dx^{\mu'} dx^{\nu'}, \tag{2}$$

defining the metric tensor $g_{\mu'\nu'}$.

Next, we define coordinates which are rotating with respect to these, with angular velocity Ω counterclockwise about the z' axis. Thus, $t = t'$, $z = z'$, $x = x' \cos \Omega t' + y' \sin \Omega t'$, and $y = y' \cos \Omega t' - x' \sin \Omega t'$. Inverting gives $x' = x \cos \Omega t - y \sin \Omega t$, and $y' = y \cos \Omega t + x \sin \Omega t$. Note that the time coordinate is identical to that in the nonrotating frame: we are not measuring time as observers rotating with the frame would (which we shall see has its advantages as well as disadvantages). Naturally, we will have an 'ergosphere' beyond $r = r_L$, where all material particles must have velocities opposing the rotation.

The differential coordinate transformation is $dx^\mu = \Lambda^\mu_{\nu'} dx^{\nu'}$, which defines the coefficients $\Lambda^\mu_{\nu'} \equiv \partial x^\mu / \partial x^{\nu'}$. The inverse transformation defines the coefficients $\bar{\Lambda}^{\mu'}_\nu \equiv \partial x^{\mu'} / \partial x^\nu$, so that $\Lambda^\mu_{\nu'} \bar{\Lambda}^{\nu'}_\sigma = \delta^\mu_\sigma$.

The metric $g_{\mu'\nu'}$ transforms as a (symmetric) second-rank covariant tensor:

$$g_{\mu\nu} = \bar{\Lambda}^{\alpha'}_\mu \bar{\Lambda}^{\beta'}_\nu g_{\alpha'\beta'} \tag{3}$$

which gives here

$$g_{\mu\nu} = \begin{pmatrix} -(1 - \Omega^2 \varrho^2) & -\Omega\, y & \Omega\, x & 0 \\ -\Omega\, y & 1 & 0 & 0 \\ \Omega\, x & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{4}$$

(Although the use of cylindrical coordinates would obviously simplify some expressions, such as this one for $g_{\mu\nu}$, we stick to rotating Cartesian spatial coordinates to avoid the added confusion cylindrical coordinates give to covariant *vs.* contravariant components of tensors.) Notice that the coefficient of dt^2 vanishes on the cylinder $\Omega^2 \varrho^2 = 1$, where $\varrho = \sqrt{x^2 + y^2}$ is the distance from the rotation axis (not to be confused with the charge density ρ). However, the determinant of $g_{\mu\nu}$ is always -1 due to the off-diagonal 'space-time' terms, so there is no mathematical difficulty in using these coordinates beyond $\varrho = r_L$. This would not be true if we had picked the times measured by local corotating observers as our time coordinate (as in Chedia *et al.* 1996), which would also have the undesirable property of having the definition of t vary with ϱ.

We shall need the inverse of $g_{\mu\nu}$. It is

$$g^{\mu\nu} = \begin{pmatrix} -1 & -\Omega\, y & \Omega\, x & 0 \\ -\Omega\, y & 1-\Omega^2 y^2 & \Omega^2 xy & 0 \\ \Omega\, x & \Omega^2 xy & 1-\Omega^2 x^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

The presence of nonzero off-diagonal terms in the metric tensor and its inverse remind us that these coordinates are not an orthogonal system, so that vectors and their associated one-forms (covectors) do not generally 'point' in the same direction. This will be shown to ultimately be the reason why the notion of separate 'electric' and 'magnetic' (spatial) vector fields runs into serious trouble in this frame.

3. Maxwell's Equations

The relativistically covariant way to discuss electrodynamics in an arbitrary coordinate system is to introduce the antisymmetric second-rank tensor **F** which has as covariant entries in the 'primed' (nonrotating) frame (Misner, Thorne, & Wheeler 1973; Weinberg 1972; Wald 1984)

$$F_{\mu'\nu'} = \begin{pmatrix} 0 & -E^{x'} & -E^{y'} & -E^{z'} \\ E^{x'} & 0 & B^{z'} & -B^{y'} \\ E^{y'} & -B^{z'} & 0 & B^{x'} \\ E^{z'} & B^{y'} & -B^{x'} & 0 \end{pmatrix} \tag{6}$$

where $E^{i'}$ are the (inertial-frame) components of the electric field $\mathbf{E}$, $B^{i'}$ are the components of the magnetic field $\mathbf{B}$, and we have assumed vacuum permeability and permittivity. The components of $F_{\mu\nu}$ in any other frame are then found in the same way as for $g_{\mu\nu}$ in equation (3):

$$F_{\mu\nu} = \bar{\Lambda}^{\alpha'}_{\mu}\bar{\Lambda}^{\beta'}_{\nu}F_{\alpha'\beta'} \tag{7}$$

We find

$$F_{\mu\nu} = \begin{pmatrix} 0 & -\tilde{E}^x & -\tilde{E}^y & -\tilde{E}^z \\ \tilde{E}^x & 0 & B^z & -B^y \\ \tilde{E}^y & -B^z & 0 & B^x \\ \tilde{E}^z & B^y & -B^x & 0 \end{pmatrix}, \tag{8}$$

where E^i and B^i are respectively defined as the projections of the electric and magnetic fields (as measured in the inertial frame) onto the new (contravariant) spatial coordinate vector basis. For example, $\mathbf{B} = B^{i'}\mathbf{e}_{i'} = B^i\mathbf{e}_i$ (at corresponding locations of the two coordinate systems; we are abusing notation), where the $\mathbf{e}_i$ are the (spatial) contravariant coordinate basis for vectors in the unprimed coordinates. That is, $\mathbf{e}_i \equiv (\partial/\partial x^i)$, holding x^j fixed for all $j \neq i$. We shall treat the E^i and B^i as numbers (rather than the contravariant components of vectors) in what follows, for clarity. $\mathbf{F}$ is the fundamental tensor of interest, while $\mathbf{E}$ and $\mathbf{B}$ are not parts of 4-vectors and so do not transform as such.

We have also defined the vector

$$\tilde{\mathbf{E}} \equiv \mathbf{E} + (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B} \tag{9}$$

which, from equation (8), seems to be the electric field in our new coordinates. However, we will also need the contravariant formulation of $\mathbf{F}$, easily found by raising the indices on equation (8) with the metric:

$$F^{\mu\nu} = \begin{pmatrix} 0 & E^x & E^y & E^z \\ -E^x & 0 & \tilde{B}^z & -\tilde{B}^y \\ -E^y & -\tilde{B}^z & 0 & \tilde{B}^x \\ -E^z & \tilde{B}^y & -\tilde{B}^x & 0 \end{pmatrix} \tag{10}$$

where we have defined

$$\tilde{\mathbf{B}} \equiv \mathbf{B} - (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{E}. \tag{11}$$

It is straightforward (and reassuring) to verify that the invariants

$$\mathbf{B} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{E} = \frac{1}{2} F^{\mu\nu} F_{\mu\nu}$$

and

$$\mathbf{E}\cdot\mathbf{B} = \frac{1}{4}F^{\mu\nu}\,{}^{*}F_{\mu\nu}$$

are reproduced by equations (8-11), where ${}^{*}F_{\mu\nu} \equiv 1/2\,\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ is the 'dual' tensor of F, and $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric fourth-rank tensor (4-form) with $\epsilon_{0123} = 1$.

Notice that equations (9) and (11) do *not* represent a local Lorentz transformation to our rotating coordinates, as our frame is not locally Lorentz (due to the global time coordinate used). Clearly, the interchanges between the 'normal' and 'tilde' versions of $\mathbf{E}$ and $\mathbf{B}$ when passing from the covariant to the contravariant representations of $\mathbf{F}$ show a problem when trying to define the fields in these coordinates. We will see from Maxwell's equations that we have no obvious way of choosing $\mathbf{E}$, $\tilde{\mathbf{E}}$, or some combination of these objects as the 'electric field' in this coordinate system (for reasons to be detailed in section 5). Some authors (Cheng 1984; Fawley *et al.* 1977; Hones & Bergeson 1965) simply follow the precedent set by Schiff (1939), who was resolving an interesting paradox involving rotating frames and Mach's principle, and who arbitrarily used the covariant choices $\tilde{\mathbf{E}}$ and $\mathbf{B}$. Schiff correctly pointed out that it is the components of the *mixed* tensor $F^{\mu}{}_{\nu}$ which dictate particle motion; we shall return to this point after deriving Maxwell's equations for the rotating frame.

Maxwell's equations in relativistically covariant form can be written (Weinberg 1972)

$$\frac{\partial(\sqrt{-g}F^{\alpha\beta})}{\partial x^{\alpha}} = 4\pi\sqrt{-g}\,J^{\beta} \tag{12a}$$

and

$$\epsilon^{\alpha\beta\gamma\delta}\frac{\partial F_{\gamma\delta}}{\partial x^{\beta}} = 0, \tag{12b}$$

where g is the determinant of $g_{\mu\nu}$ (simply -1 here), and J is the current density 4-vector. In the inertial frame, we have $J'^{\alpha} = (\rho', \mathbf{j}')$ which transforms to $J^{\alpha} = (\rho, \mathbf{j})$, where $\rho = \rho'$, and $\mathbf{j} \equiv \mathbf{j}' - \rho(\mathbf{\Omega}\times\mathbf{r})$. Note that the convection of charge due to the rotation alters the effective current density.

Equation (12a) produces

$$\nabla \cdot \mathbf{E} = 4\pi\rho \tag{13a}$$

and

$$\nabla \times \tilde{\mathbf{B}} = 4\pi\,\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}, \tag{13b}$$

while equation (12b) gives

$$\nabla \cdot \mathbf{B} = 0 \tag{13c}$$

and

$$\nabla \times \tilde{\mathbf{E}} + \frac{\partial \mathbf{B}}{\partial t} = 0. \tag{13d}$$

Note the appearance of both 'normal' and 'tilde' components of **E** and **B**. Equations (13) constitute Maxwell's equations in this frame. Here, ∇ is the usual 3-dimensional gradient operator, if rotating Cartesian coordinates are used.

We combine equations (13a) and (13b) to get an equation for charge conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \tag{14}$$

Equations (13) and (14) suggest that there is no difficulty in interpreting ρ and $\mathbf{j}$ as charge and current densities in this frame. However, J^α is (unlike **E** and **B**) a genuine 4-vector, with covariant components $J_\alpha = (-\rho(1 - \Omega^2\varrho^2) + (\mathbf{\Omega} \times \mathbf{r}) \cdot \mathbf{j}, \mathbf{j}')$.

4. Geodesics and Lorentz Force

To explore particle dynamics, we use the geodesic equation, modified by the Lorentz force when necessary. First, we find the Christoffel symbols from the metric:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta} + \frac{\partial g_{\nu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right). \tag{15}$$

The only nonvanishing ones are found to be $\Gamma^x_{tt} = -\Omega^2 x$, $\Gamma^y_{tt} = -\Omega^2 y$, $\Gamma^x_{yt} = \Gamma^x_{ty} = -\Omega$, and $\Gamma^y_{xt} = \Gamma^y_{tx} = \Omega$.

We now define the 4-velocity of a massive particle to be

$$U^\mu = \frac{dx^\mu}{d\tau} \tag{16}$$

where τ is the proper time of the particle. Notice that this gives $U^\mu U_\mu = g_{\mu\nu} U^\mu U^\nu = -1$, by the definition of proper time. A particle subject to no external forces travels along geodesics, which are the solutions of

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} U^\nu U^\lambda = 0. \tag{17}$$

The temporal equation is trivial:

$$\frac{d^2 t}{d\tau^2} = 0,$$

giving

$$\frac{dt}{d\tau} = \mathrm{constant} \equiv \gamma.$$

We note that

$$\frac{d}{d\tau} = \frac{dt}{d\tau}\frac{d}{dt} = \gamma \frac{d}{dt},$$

so that $U^\mu = (\gamma, \gamma \mathbf{v})$, where $\mathbf{v} = d\mathbf{x}/dt$. Plugging this into $g_{\mu\nu} U^\mu U^\nu = -1$ gives, after rearranging,

$$\gamma = \frac{1}{\sqrt{1 - v'^2}} \tag{18}$$

where $\mathbf{v}' \equiv \mathbf{v} + \mathbf{\Omega} \times \mathbf{r}$, the particle's velocity as measured in the *nonrotating* frame of reference. Thus, γ (which is conserved in the absence of external forces) is just the particle energy in the nonrotating frame (in units of mc^2). Note, however, that γ is not equal to $1/\sqrt{1 - v^2}$, which is fortunate since $v > 1$ is possible in this frame.

The spatial geodesic equations give (after cancelling a common γ)

$$\frac{d\mathbf{v}}{dt} = -\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) - 2(\mathbf{\Omega} \times \mathbf{v}).$$

This is just the familiar expression for the Coriolis and centrifugal forces in a rotating frame. They are still valid relativistically, even beyond the light cylinder.

When external forces are involved, we need only to introduce all forces (per unit mass) as 4-vectors f^α, and insert them on the right hand side of the geodesic equations in a covariant manner: $dU^\alpha/d\tau + \Gamma^\alpha_{\mu\nu}U^\mu U^\nu = f^\alpha$.

The Lorentz force is given by the contraction of $F^{\mu\nu}$ with the 4-velocity U^α:

$$f^\alpha = \frac{q}{m} g_{\mu\nu} F^{\alpha\mu} U^\nu \tag{19a}$$

or equivalently by using the 'mixed' representation of $\mathbf{F}$:

$$\frac{dU^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} U^\nu U^\lambda = \frac{q}{m} F^\mu{}_\nu U^\nu, \tag{19b}$$

where q is the charge on the particle, and m its mass. The mixed tensor is here

$$F^\mu{}_\nu = \begin{pmatrix} \mathbf{E}\cdot(\mathbf{\Omega}\times\mathbf{r}) & E^x & E^y & E^z \\ \tilde{E}^x - E^x\Omega^2\varrho^2 & \Omega y E^x & \tilde{B}^z - \Omega x E^x & -\tilde{B}^y \\ \tilde{E}^y - E^y\Omega^2\varrho^2 & -B^z - \Omega x E^x & -\Omega x E^y & \tilde{B}^x \\ \tilde{E}^z - E^z\Omega^2\varrho^2 & B^y & -B^x & 0 \end{pmatrix}. \tag{20}$$

Substituting this Lorentz force into the geodesic equations gives

$$\frac{d\gamma}{dt} = \frac{q}{m}\Big[\mathbf{v} + (\mathbf{\Omega}\times\mathbf{r})\Big]\cdot\mathbf{E} = \frac{q}{m}\mathbf{v}'\cdot\mathbf{E} \tag{21}$$

for the temporal equation, and

$$\frac{d(\gamma\mathbf{v})}{dt} = \gamma\Big[-\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}) - 2\,\mathbf{\Omega}\times\mathbf{v}\Big] + \frac{q}{m}\Big[\tilde{\mathbf{E}} + \mathbf{v}\times\mathbf{B} - (\mathbf{v}'\cdot\mathbf{E})(\mathbf{\Omega}\times\mathbf{r})\Big] \tag{22}$$

for the spatial equations. Note the reappearance of γ in the Coriolis and centrifugal forces, since γ is no longer conserved.

We call attention to the term

$$(\mathbf{\Omega}\times\mathbf{r})\cdot\mathbf{E}$$

in equation (21). It shows that a charged particle may gain or lose inertia even when undergoing pure corotation, provided there is a component of $\mathbf{E}$ in the direction of $\boldsymbol{\Omega} \times \mathbf{r}$. This is obvious when considering particle motion in the nonrotating coordinates, but is easy to forget when passing to the rotating frame, where the 'motion' becomes hidden.

We also see that the Lorentz force takes on a peculiar form in our rotating frame. The term $\tilde{\mathbf{E}} + \mathbf{v} \times \mathbf{B}$ seems to lend credibility to naming $\tilde{\mathbf{E}}$ and $\mathbf{B}$ as the electric and magnetic fields, but note that we could have also have written it as $\mathbf{E} + \mathbf{v}' \times \mathbf{B}$.

What seems to be absent in existing literature is the final term in the Lorentz force, which can also be written as

$$-(\boldsymbol{\Omega} \times \mathbf{r})\frac{d\gamma}{dt}.$$

Note that the direction of this force is against (with) the rotation for a particle which is gaining (losing) energy, betraying its origin as an inertial 'relativistic mass' effect.

5. Interpretation

We have already seen in equation (14) that ρ and $\mathbf{j}$ can be naturally identified with the charge and current densities in this frame; we wish to make similar identifications of the electric and magnetic fields. The standard technique from special relativity determines the electric field by contracting U^α with the mixed representation of $\mathbf{F}$:

$$E^\alpha = (0, \mathbf{E}) \equiv F^\alpha{}_\beta U^\beta,$$

but this doesn't work here due to the nonzero component $F^t{}_t$ in equation (20), which is also responsible for the $(\boldsymbol{\Omega} \times \mathbf{r}) \cdot \mathbf{E}$ term in equation (21). (It is interesting to note that it is this 'azimuthal' component of $\mathbf{E}$ for which $\mathbf{E}$ and $\tilde{\mathbf{E}}$ always agree.) Clearly, no prescription for 'electric' and 'magnetic' fields will reproduce the Lorentz force as we are most familiar with it.

The root of the problem is seen by considering equation (19b), which says that the Lorentz force modifies geodesic motion by 'rotating' the tangent U^α to a particle's worldline away from its parallel-transported value (along the worldline). These 'rotations,' by (the 'unprimed' version of) the metric equation (2), are restricted to satisfy the equation $g_{\mu\nu}U^\mu\, dU^\nu/dt = 0$. In the flat Minkowski metric of special relativity, these rotations are just the directions of Lorentz boosts and spatial rotations. We naturally identify forces which produce spatial rotations of U^α as magnetic in nature, and those which boost U^α as electric (for electrically charged particles). In this rotating reference frame, timelike and spacelike components are not orthogonal to one another, so boosts and rotations necessarily become mixed. Indeed, infinitesimal Lorentz boosts and rotations transform in exactly the same way as do $\mathbf{E}$ and $\mathbf{B}$, respectively. We conclude from this that any reference frame with non-diagonal metric tensor $g_{\mu\nu}$ will suffer the loss of precise definitions of electric and/or magnetic fields.

Nowhere is this more apparent than in equation (21), which allows a charged particle's inertia to change provided $(\mathbf{\Omega} \times \mathbf{r}) \cdot \mathbf{E}$ is nonzero, even when the particle is at rest in the corotating frame. Conservation of angular momentum demands that this exerts a torque on the source of the fields (*e.g.*, the neutron star), which can alter the frequency of rotation. This phenomenon could be put to beneficial use in pulsar models. The 'braking index' of pulsars, defined as

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}$$

(where the dots denote time derivatives) has, when observable, always been found smaller than the canonically predicted value $n = 3$. (This is predicted as a *lower* limit when the torque is from simple multipole radiation — see, for example, Kaspi *et al.* 1994 and references therein.) Presently, the discrepancy is understood as being due to particle outflow along lines of $\mathbf{B}$, or due to more complicated effects. We see, however, that even the corotating portion of a magnetosphere, with $\mathbf{B} \cdot \mathbf{E} = 0$, can alter the rotation rate by doing work on charged particles (if $\mathbf{E} \cdot (\mathbf{\Omega} \times \mathbf{r}) \neq 0$).

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