ALMA Memo. No. 479

Requirements for Subreflector and Feed Positioning for ALMA Antennas

Bryan J. Butler
National Radio Astronomy Observatory

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Abstract

This memo presents some calculations related to requirements on subreflector and feed positioning for the ALMA antennas. In order to keep gain loss at less than 1%, the axial (vertical) positioning error in the subreflector should be less than about 0.09 λ, and in the feed should be less than about 0.9 λ. The lateral positioning error for the subreflector should be less than about 0.45 λ, and for the feed should be less than about 10 λ. The rotational (tilt) error of the subreflector should be less than about 5.5 $\lambda_{\rm mm}$ arcminutes if the rotation is about the vertex of the subreflector. If the rotation is about the prime focus, then this error can be much larger: about $\sqrt{\lambda_{\rm mm}}$ degrees.

1 Introduction

Incorrect positioning of the subreflector and feeds of the ALMA antennas will result in a loss in gain. There are other more subtle effects (like changing the primary beam shape and the phase across the primary beam, which are issues for mosaicing), but these are not treated in this memo - only the loss in gain from errors in subreflector and feed positioning.

2 Theory

This study is based nearly completely on an unpublished note written by John Ruze in 1969 titled "Small Displacements in Parabolic Reflectors." A similar treatment of this problem, but for displacements of the feeds for the MMA antennas, is in Shillue (1997). An unpublished note by James Lamb ("Verification of Ruze Formulas By Comparison with Ray-Tracing", dated 2001-May-09) gives a correction of the formulas in Ruze's note, making sure that the sign conventions are always consistent, and not normalizing the path error to the on-axis error (this allows for the effects of multiple offsets to be added in the aperture plane all at once). This note is included here as Appendix A. Using polar coordinates in the aperture plane r and ϕ, the phase errors resulting from incorrect feed or subreflector positioning are denoted $\delta(r, \phi)$. Given the subreflector offsets, the form of $\delta(r, \phi)$ can be explicitly written, and for a Cassegrain reflector (as for ALMA) are shown in Table 1 (taken from Lamb 2001).

The geometrical quantities in Table 1 are:

$\theta_p =$ angle between optical axis and a ray from the feed to the subreflector;

$\theta_f =$ angle between optical axis and a ray from the subreflector to the main reflector;

$M =$ antenna magnification;

$c - a =$ distance from primary focus to subreflector surface along the optical axis.

Table 1: Form of Phase Errors for Given Subreflector Offsets.

type	$\delta(r,\phi)$
feed axial displacement (Δz_f)	$-\dfrac{2\pi\Delta z_f}{\lambda}\cos\theta_f$
feed lateral displacement (Δr_f)	$-\dfrac{2\pi\Delta r_f}{\lambda}\sin\theta_p\cos\phi$
subreflector axial displacement (Δz_s)	$\dfrac{2\pi\Delta z_s}{\lambda}\left(\cos\theta_p + \cos\theta_f\right)$
subreflector lateral displacement (Δr_s)	$-\dfrac{2\pi\Delta r_s}{\lambda}\left[\sin\theta_p - \sin\theta_f\right]\cos\phi$
subreflector rotation ($\Delta\alpha$)	$-\dfrac{2\pi\Delta\alpha\ \lvert c-a\rvert}{\lambda}\left[\sin\theta_p + M\sin\theta_f\right]\cos\phi$

For the ALMA antennas, M is 20, and to first order, we can assume that $c - a = f/M$ for focal length f, which is 4.8 m for the ALMA antennas (Lamb 1999).

Given the phase error across the aperture, the loss in gain is calculated via (equation 3 of Ruze 1969):

$$\frac{G}{G_o} = 1 - \overline{\delta^2} + \overline{\delta}^2 \quad , \tag{1}$$

where

$$\overline{\delta^2} = \frac{\displaystyle\int_0^{2\pi}\int_0^1 f(r,\phi)\,\delta^2(r,\phi)\,r\,dr\,d\phi}{\displaystyle\int_0^{2\pi}\int_0^1 f(r,\phi)\,r\,dr\,d\phi} \quad , \tag{2}$$

and

$$\overline{\delta} = \frac{\displaystyle\int_0^{2\pi}\int_0^1 f(r,\phi)\,\delta(r,\phi)\,r\,dr\,d\phi}{\displaystyle\int_0^{2\pi}\int_0^1 f(r,\phi)\,r\,dr\,d\phi} \quad . \tag{3}$$

The function $f(r,\phi)$ is the feed illumination function, which to first order can be assumed to be: $f(r) = 1 - ar^2$. For a 12dB Gaussian taper on the feed illumination, $a = 0.75$, which is the value used in this study.

Before calculating the above integrals, a best-fit linear phase plane across the aperture must be subtracted (this is the equivalent of what offset pointing does for you). This is done by calculating the phase across the aperture and doing a least squares fit for the parameters of the plane, and then subtracting that best-fit plane from the raw calculated phases before calculating the integrals.

Note that the situation for the ALMA antennas is more complicated than this, because the feeds are not all exactly on-axis. To account for the displacement from the optical axis, the subreflector will be shifted laterally to refocus the beam onto the slightly off-axis feeds (Lamb et al. 2001). The analysis here is still correct to first order, though, and allows for limits to be placed on the deviation of the subreflector or feed from its *nominal* position, whether that is on-axis focus, or slightly offset to focus off-axis.

3 Results

The resultant curves for loss of gain are shown in Figures 1 through 5, as functions of the positioning errors. In order to have the loss in gain be less than 1% from the axial positioning errors, the error in the subreflector should be less than about 0.09 λ, and in the feed should be less than about 0.9 λ. The lateral positioning error for the subreflector should be less than about 0.45 λ, and for the feed should be less than about 10 λ. The rotational (tilt) error of the subreflector should be less than about 5.5 λ_{mm} arcminutes.

3.1 A Comparison

An unpublished note by James Lamb ("Secondary Mirror Positional Tolerances", dated 1999-Jan-29), contains formulae for the effective rms introduced by subreflector misalignments. This note is included here as Appendix B. One slight difference in that work and this one is that the assumed illumination was different. James assumed uniform illumination, while this work uses an illumination function which approximates a Gaussian taper with an edge taper value of -12 dB. So slightly different results should be expected, but they should be the same to first order. In any case, in order to compare the results here with those, it is necessary to first derive the relationship between effective rms due to optics misalignment and gain loss.

Define the aperture efficiency as:

$$\eta_a = \eta_o e^{-(4\pi\sigma'/\lambda)^2} \quad , \tag{4}$$

for wavelength λ and total rms σ'. Assume that the total rms is made up of two terms: a main dish component (σ) and an optics misalignment component (ϵ) and that they add in quadrature for the total:

$$\sigma'^2 = \sigma^2 + \epsilon^2 \quad . \tag{5}$$

Then the ratio of the aperture efficiency with no optics misalignment to that with the optics misalinment is:

$$R = e^{-(4\pi\epsilon/\lambda)^2} \quad . \tag{6}$$

Since ϵ is small, expand the exponential and drop all higher order terms:

$$R = 1 - \left(\frac{4\,\pi\,\epsilon}{\lambda}\right)^2 \quad . \tag{7}$$

Now, assuming that $R = 0.99$, as in the treatment above, leaves:

$$\epsilon \sim 0.008\ \lambda \quad . \tag{8}$$

3.1.1 Subreflector Lateral Displacement

James' unpublished note gives:

$$\epsilon = a_1\ \Delta x \quad , \tag{9}$$

where $a_1 = 19.1\ \mu\mathrm{m\ mm}^{-1}$. For comparison with the results of this memo, substitute $\Delta x = 0.45\lambda$, and $\epsilon = 0.008\lambda$:

$$0.008\lambda = a_1'\ 0.45\ \lambda \quad , \tag{10}$$

yielding $a_1' = 17.8\ \mu\mathrm{m\ mm}^{-1}$. This is good agreement.

3.1.2 Subreflector Axial Displacement

James' unpublished note gives:

$$\epsilon = a_2\ \Delta z \quad , \tag{11}$$

where $a_2 = 80.7\ \mu\mathrm{m\ mm}^{-1}$. For comparison with the results of this memo, substitute $\Delta x = 0.09\lambda$, and $\epsilon = 0.008\lambda$:

$$0.008\lambda = a_2'\ 0.09\ \lambda \quad , \tag{12}$$

yielding $a_2' = 88.9\ \mu\mathrm{m\ mm}^{-1}$. Again, pretty good agreement.

3.1.3 Subreflector Rotation

James' unpublished note gives:

$$\epsilon = a_3 \; \Delta\theta^2 \quad , \tag{13}$$

where $a_3 = 8.16$ μm deg^{-2}. For comparison with the results of this memo, substitute $\Delta\theta = 91.7\lambda$ (which is $5.5\lambda_{\rm mm}$ arcmin, converted to degrees), and $\epsilon = 0.008\lambda$:

$$0.008\lambda = a_3' \; (91.7 \; \lambda)^2 \quad , \tag{14}$$

yielding $a_3' = 0.951/\lambda$ μm deg^{-2}. Here, there are two immediately apparent problems - the proportionality is different (the dependence on wavelength), and the magnitude is significantly different (order of magnitude at $\lambda = 1$ mm: James' result would allow for a rotational error as large as ~ 1 deg at 1 mm, while the result here is that it can only be ~ 6 arcmin at that wavelength). Reducing James' formula to the allowable error as a function of wavelength gives: $\Delta\theta \sim \sqrt{\lambda_{\rm mm}}$ degrees. This difference can be attributed entirely to the assumption of where the rotational center is. Ruze's formula assumes that the rotation is about the vertex of the subreflector, while James assumes that the rotation is about the prime focus. These are two end-members in a sense - best and worst case scenarios. Until the design for the subreflector motion mechanism is finalized, it is difficult to say what the final requirement will be on rotation of the subreflector. At the time the design is finalized and the point about which effective rotations occur is known, a new study should be completed and this issue can be settled. It should be noted, however, that rotations about any other point than the prime focus can be decomposed into a lateral translation and a rotation about the prime focus. For this reason, if the subreflector measurement and adjustment scheme first fixes the lateral translation and then the subreflector rotation, the effective rotation error will be about the prime focus. Furthermore, rotations about non-prime focus locations cause wavefront aberrations which do not add in quadrature with the other aberrations. These two reasons are why James took the approach that he did, and because of this his numbers will likely turn out to be closer to the truth.

Acknowledgements

Discussions with Peter Napier helped immensely. Obviously James Lamb's documents (Appendices A and B) also helped considerably, as did discussions with him.

References

Lamb, J.W., A. Baryshev, M.C. Carter, L.R. D'Addario, B.N. Ellison, W. Grammer, B. Lazareff, Y. Sekimoto, & C.Y. Tham, ALMA Receiver Optics Design, MMA Memo 362, 2001

Lamb, J.W., Optimized Optical Layout for MMA 12-m Antennas, MMA Memo 246, 1999

Shillue, B., Gain Degradation in a Symmetrical Cassegrain Antenna Due to Laterally Offset Feeds, MMA Memo 175, 1997

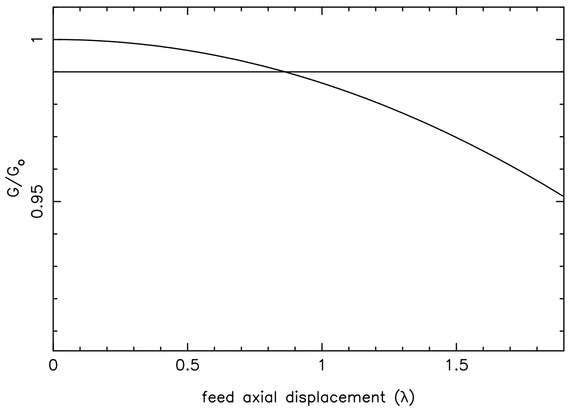

Figure 1: Loss of gain as a function of feed axial offset, in wavelengths.

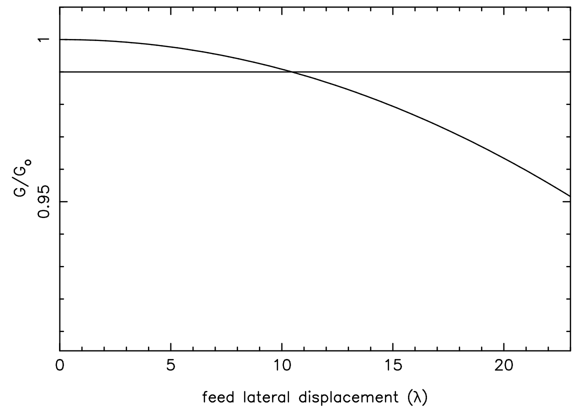

Figure 2: Loss of gain as a function of feed lateral offset, in wavelengths.

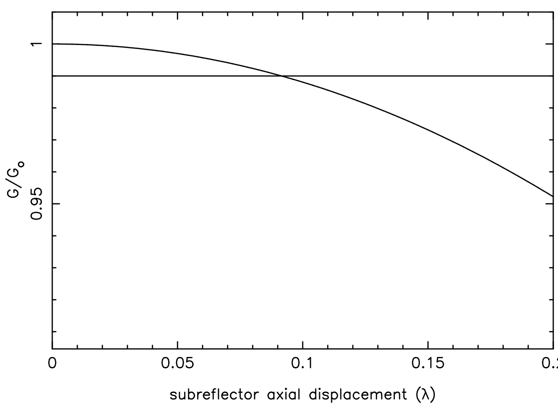

Figure 3: Loss of gain as a function of subreflector axial offset, in wavelengths.

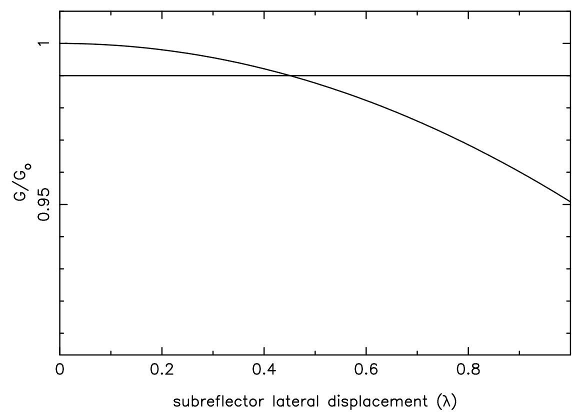

Figure 4: Loss of gain as a function of subreflector lateral offset, in wavelengths.

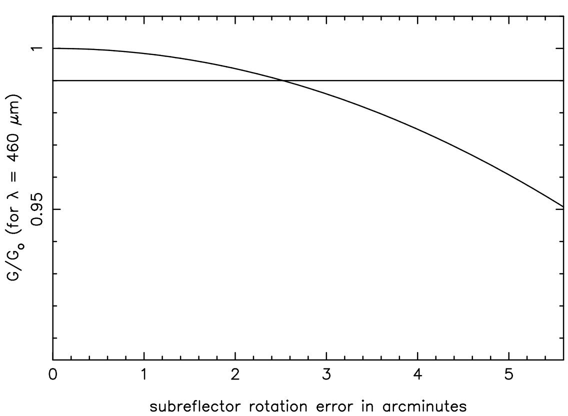

Figure 5: Loss of gain as a function of subreflector rotation error, in arcminutes, for an observing frequency of 650 GHz ($\lambda \sim 460$ μm).

Appendix A

Verification of Ruze Formulas By Comparison with Ray-Tracing

James W Lamb
OVRO, Caltech
2001-05-29

1. Introduction

In many instances it is expedient to evaluate the effective path error due to displacements of the feed or secondary mirror in closed form. Ruze [1] has derived some simple formulas for this purpose. When these are to be used with other calculations, such as finite-element-analysis (FEA) it is important that the coordinate systems and sign conventions are clear and consistent. To verify the form and sign of these formulas we compare them with results from ray tracing in Mathcad. Corrections are made to the signs of the original formulas, and it is pointed out that the normalization to the path error at the center of the aperture can lead to confusion and incorrect results if care is not taken.

2. Geometry

The geometry is defined in Figure 1. Two coordinate systems will be used, a cylindrical one with the z-axis along the boresight direction, and a rectangular system which follows the ALMA convention for the antennas [2]. The cylindrical system results in simpler equations, but the rectangular system may be easier to implement in an FEA analysis. Formulas in both systems are provided.

The antenna has a diameter D, a focal length f, and a magnification M.

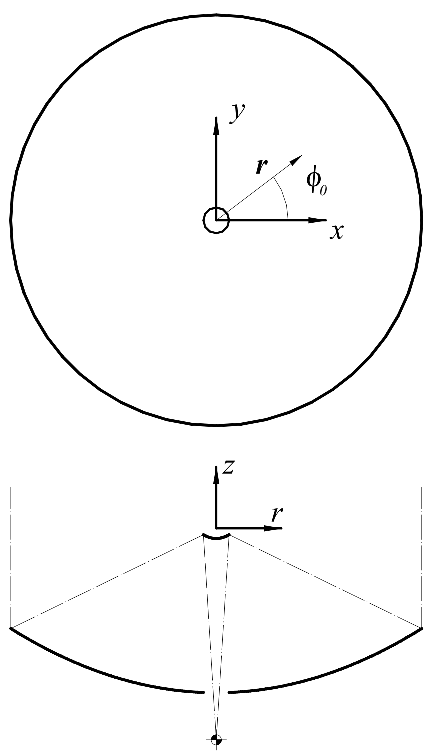

Figure 1. Geometry and coordinate system. Third angle projection is used, so the upper view is from above the antenna pointed at zenith, and the lower view is from the rear of the antenna. The antenna rotates about the negative x–axis down towards the horizon position.

3. Cylindrical Coordinates

The simplest forms of the equations are given in terms of the cylindrical coordinates (r, ϕ, z). Table I presents the error in this coordinate system. The following points should be noted:

- The path error is the *actual path length* minus the *ideal path length*. That is, if the formula gives a *positive* number, the path length from the antenna focus to the antenna aperture plane is *increased*. (The original Ruze paper was inconsistent in its sign convention.)
- Axial displacements are in the positive z direction.
- Radial displacements are along a vector $\boldsymbol{r}$ at an angle ϕ_0 to the x–axis.
- Tilts of the secondary mirror are about a vector parallel to $\boldsymbol{z} \times \boldsymbol{r}$ and passing through the *secondary mirror vertex*. Rotations about other points are a combination of this plus a lateral shift.
- The formulas are often renormalized by setting the path error to zero at the center of the aperture. Table I shows both the normalized and un-normalized equations, where these differ. The un-normalized ones can be directly compared with the ray-tracing results.

Table I. Path length error in terms of cylindrical coordinates.

Position Error	Actual path error	Normalized to zero on axis
Feed axial displacement by Δz_f	$-\Delta z_f \cos(\theta_f)$	$\Delta z_f \left(1-\cos(\theta_f)\right)$
Feed lateral displacement by Δr_f at angle ϕ_0	$-\Delta r_f \sin(\theta_f)\cos(\phi-\phi_0)$	
Secondary mirror axial displacement by Δz_s	$\Delta z_s \left(\cos(\theta_p)+\cos(\theta_f)\right)$	$\Delta z_s \left(\left(\cos(\theta_p)-1\right)+\left(\cos(\theta_f)-1\right)\right)$
Secondary mirror lateral displacement by Δr_s at angle ϕ_0	$-\Delta r_s \left(\sin(\theta_p)-\sin(\theta_f)\right)\cos(\phi-\phi_0)$	
Secondary mirror tilt around vertex $\Delta\alpha$	$-\Delta\alpha(c-a)\left(\sin(\theta_p)+M\sin(\theta_f)\right)\cos(\phi-\phi_0)$	

Where the angles θ_p and θ_f are defined by

$$\sin(\theta_p)=\frac{\frac{r}{f}}{1+\left(\frac{r}{2f}\right)^2} \qquad \sin(\theta_f)=\frac{\frac{r}{Mf}}{1+\left(\frac{r}{2Mf}\right)^2}$$

The above equations were compared directly with a Mathcad ray-tracing analysis which ensures that the sign convention is consistent and the magnitudes of the results are correct. The actual expressions implemented in the Mathcad document are (with F = Mf):

$$\theta_p := 2\cdot\mathrm{atan}\left(\frac{r}{2\cdot f}\right) \qquad \theta_f := 2\cdot\mathrm{atan}\left(\frac{r}{2\cdot F}\right)$$

$$p_1 := -\Delta z_f\cdot\cos(\theta_f) \qquad p_2 := -\Delta r_f\cdot\sin(\theta_f)$$

$$p_3 := \Delta z_s\cdot\left(\cos(\theta_p)+\cos(\theta_f)\right) \qquad p_4 := -\Delta r_s\cdot\left(\sin(\theta_p)-\sin(\theta_f)\right)$$

$$p_5 := -\alpha_y\cdot(c-a)\cdot\left(\sin(\theta_p)+M\cdot\sin(\theta_f)\right) \qquad p := p_1+p_2+p_3+p_4+p_5$$

The following graphs show the comparison with the ray-tracing results.

$\Delta z_f = 10\,\mathrm{mm}$

$\Delta r_f = 0\,\mathrm{mm}$

$\Delta z_s = 0\,\mathrm{mm}$

$\Delta r_s = 0\,\mathrm{mm}$

$\alpha_y = 0\,\mathrm{deg}$

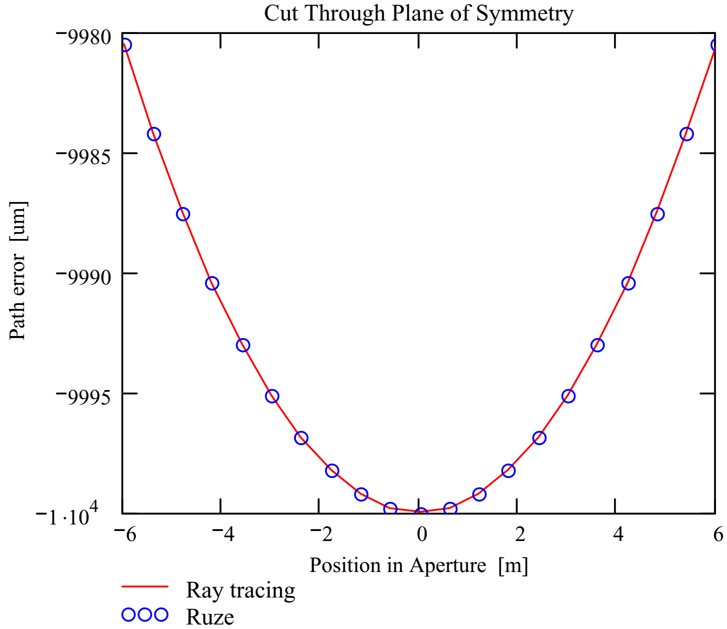

Figure 2. Axial movement of feed.

$\Delta z_f = 0\,\mathrm{mm}$

$\Delta r_f = 10\,\mathrm{mm}$

$\Delta z_s = 0\,\mathrm{mm}$

$\Delta r_s = 0\,\mathrm{mm}$

$\alpha_y = 0\,\mathrm{deg}$

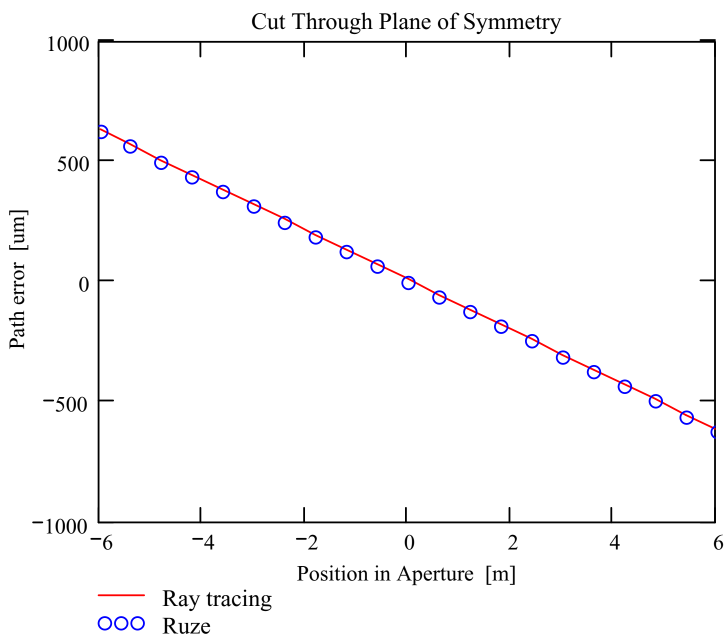

Figure 3. Lateral feed shift.

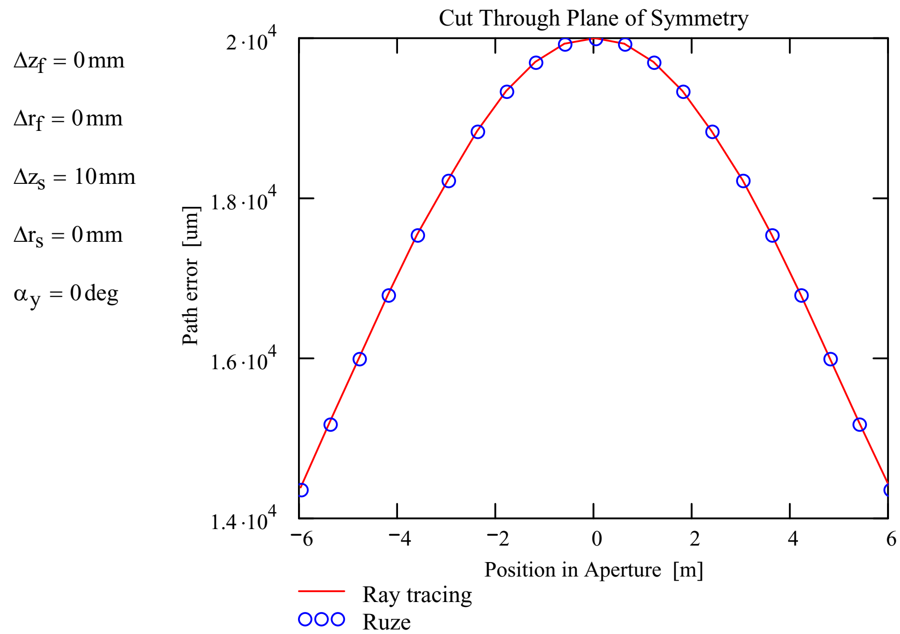

Figure 4. Axial movement of secondary mirror.

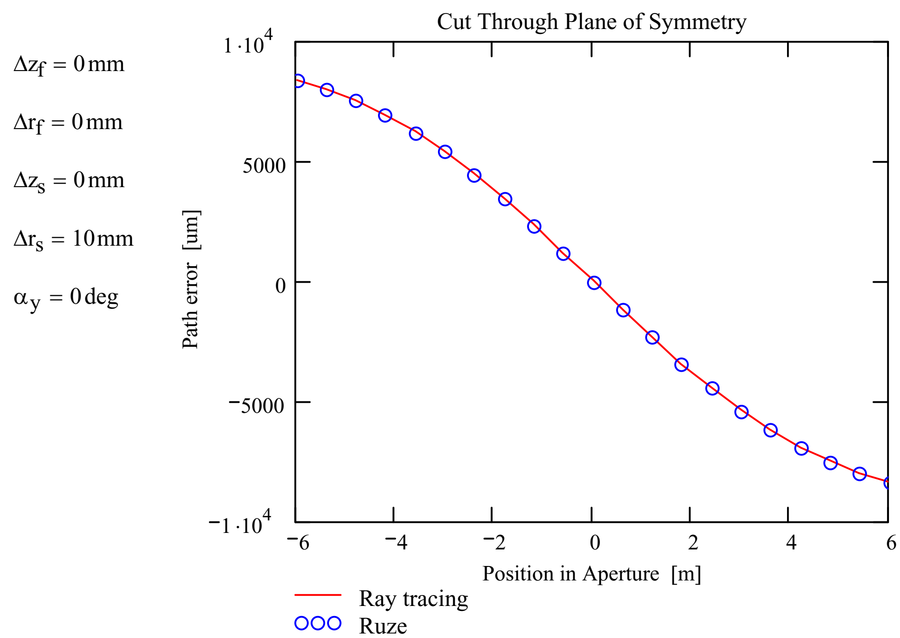

Figure 5. Lateral movement of secondary mirror.

$\Delta z_f = 0\,\mathrm{mm}$

$\Delta r_f = 0\,\mathrm{mm}$

$\Delta z_s = 0\,\mathrm{mm}$

$\Delta r_s = 0\,\mathrm{mm}$

$\alpha_y = 1\,\mathrm{deg}$

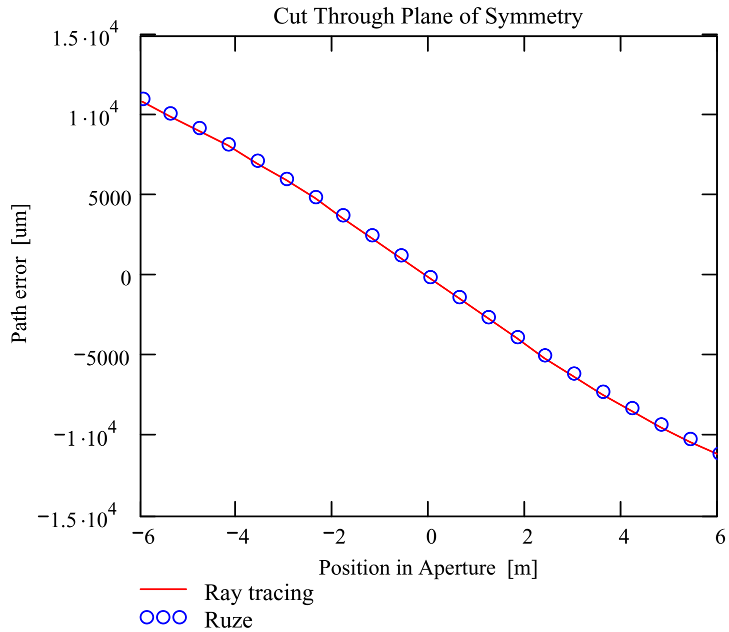

Figure 6. Tilt of secondary mirror.

As seen in Figure 2–Figure 6 the equations do agree well with the numerical results, and in particular the signs are consistent. The calculations also verify that the path errors may be simply added to give the same results as ray tracing.

4. Cartesian Coordinates

In Cartesian coordinates the formulas take the forms shown in Table II. To compare with the previous section, the lateral displacements and tilts are decomposed into two components parallel to the *x*– and y–axes:

$$\Delta x_f = \Delta r_f \cos(\phi_0)$$
$$\Delta y_f = \Delta r_f \sin(\phi_0)$$
$$\Delta x_s = \Delta r_s \cos(\phi_0)$$
$$\Delta x_s = \Delta r_s \sin(\phi_0)$$
$$\Delta\alpha_x = -\Delta\alpha_s \sin(\phi_0)$$
$$\Delta\alpha_y = \Delta\alpha_s \cos(\phi_0)$$

Note that the rotations $\Delta\alpha_x$ and $\Delta\alpha_y$ are taken as positive rotations around the *x*– and *y*–axes respectively. The *axis of rotation* passes through the *vertex* of the secondary mirror.

Table II. Path length error in terms of cylindrical coordinates.

Position Error	Actual path error	Normalized to zero on axis
Feed axial displacement by Δz_f	$-\Delta z_f \cos(\theta_f)$	$\Delta z_f \left(1-\cos(\theta_f)\right)$
Feed lateral displacements by Δx_f, Δy_f	$-\left(\Delta x_f \cos(\phi)+\Delta y_f \sin(\phi)\right)\sin(\theta_f)$	
Secondary mirror axial displacement by Δz_s	$\Delta z_s\left(\cos(\theta_p)+\cos(\theta_f)\right)$	$\Delta z_s\left(\left(\cos(\theta_p)-1\right)+\left(\cos(\theta_f)-1\right)\right)$
Secondary mirror lateral displacements by Δx_s, Δy_s	$-\left(\Delta x_s \cos(\phi)+\Delta y_s \sin(\phi)\right)\left(\sin(\theta_p)-\sin(\theta_f)\right)$	
Secondary mirror tilts by $\Delta\alpha_x$, $\Delta\alpha_y$	$\left(\Delta\alpha_x \sin(\phi)-\Delta\alpha_y \cos(\phi)\right)\left(c-a\right)\left(\sin(\theta_p)+M\sin(\theta_f)\right)$	

Where

$$\sin(\theta_p)=\frac{\frac{r}{f}}{1+\left(\frac{r}{2f}\right)^2} \qquad \sin(\theta_f)=\frac{\frac{r}{Mf}}{1+\left(\frac{r}{2Mf}\right)^2}$$

$$r=\sqrt{x^2+y^2} \qquad \phi=\tan^{-1}(y/x)$$

5. Discussion

The formulas presented by Ruze [1] have been shown to be accurate for small displacements of the secondary focus or secondary mirror in the ALMA antennas once the correct signs are used. It is critical that when the results are combined with FEA the signs must be consistent since the effects may be additive. An example is where the primary mirror opens out under gravity and pulls the secondary mirror in closer to the primary. The focus of the primary moves out from the antenna, but the secondary moves in opposite direction exacerbating the effect.

Another pitfall is that the formulas are usually re-normalized to refer them to the path length at the center of the antenna aperture (This affects only the expressions for displacements along the antenna axis). This can be confusing since the sign of the actual path change given by $-\Delta z_f \cos(\theta_f)$ has the opposite sign from the normalized version $\Delta z_f\left(1-\cos(\theta_f)\right)$, though the shape of the function is the same. If the equations are to be applied to part of the primary mirror (to find the effect of panel displacements, for example) then the un-normalized formulas must be used. Another case where the distinction is important is in the estimation of the *phase* errors induced. We would recommend, therefore, that the un-normalized forms should always be used. This will not affect the calculation of the contribution to the wavefront or effective surface error.

References

[1] J. Ruze, “Small displacements in parabolic reflectors,” Unpublished, Feb. 1969.
[2] H. Riewaldt, “Basic antenna definitions,” NRAO, ALMA-US ICD No. 11, Jan. 2000.

Appendix B

Secondary Mirror Positional Tolerances

James W Lamb

Abstract— **Tolerances for the positioning of the secondary mirror are derived from the associated aberrations. The optimum location of the secondary will be determined by radiometric measurements at regular intervals. It is the precision of these measurements that should limit the accuracy, rather than the mechanics of the translation stage. The translation stage for the secondary should have a resolution of ~5–10 μm, and a linearity of 0.2 % on each axis.**

I. INTRODUCTION

The secondary mirror will have a focus-translation stage allowing focus optimization during operation. Requirements on the positioning accuracy are set by the contributions allocated to the surface accuracy, pointing and phase error budgets. To understand how to specify these it is necessary to know exactly how they enter into these budgets.

Since the secondary mirror has circular symmetry there are five parameter which need to be specified, three translational and two rotational. The translation errors are Δx and Δy orthogonal to the optical axis, and Δz parallel to it. Tilt errors are denoted by $\Delta\theta_x$ and $\Delta\theta_y$ around the *x*-axis and *y*-axis respectively.

To determine the effect of these errors ray tracing was used to calculate the wavefront error. This is slightly more accurate than some of the analytical formulas which are valid only for large focal ratios. Results quoted here assume uniform aperture illumination, but including a 12-dB taper does not change the results significantly. The antenna parameters used in the calculations are given in Table I

The results will be given in terms of an effective surface error (half wavefront error) which would be added in quadrature to other components of the surface accuracy budget.

TABLE I: ANTENNA PARAMETERS USED IN THE TEXT

Parameter	Symbol	Value
Primary diameter	D	12.00 m
Primary focal length	f	4.8 m
Secondary diameter	d	750 mm
Magnification	M	20

January 29, 1999

Owens Valley Radio Observatory, California Institute of Technology, Big Pine, CA 93513, USA

II. SETTING PROCEDURE

The antenna primary mirror will be set up initially using a theodolite and tape. This should achieve a surface accuracy of ~100 μm, and reference the optical axis to the gravitational vertical. The surface will then be measured using a holography receiver mounted at the prime focus and adjusted according to the surface errors derived from this. This effectively defines the location of the prime focus relative to the holography feed phase center.

Once the secondary mirror has been installed it will be aligned so that it is perpendicular to the optical axis. Since there is no provision for any tilting mechanism, this will be done with shims, manual adjustment screws, or by machining. The axial and lateral (radial) focus positions will then be determined from astronomical observations. It will be assumed for this analysis that these measurements are made at a wavelength $\lambda = 1$ mm. At shorter wavelengths the radiometric sensitivity will be lower, but the required precision as a fraction of the wavelength will be less. The wavelength is therefore not critical.

Pointing and focus will be checked every 30 min and, if required, the focus optimized to the accuracy of these measurements. The translation stage needs to be accurate enough to track focus changes between these measurements. Periodic calibrations of phase will be made at intervals of 3 min on sources offset on the sky by up to 2°. If the focus needs to be changed between these calibrations, the *z*-translation accuracy needs to be within the pathlength error budget.

III. GRAVITATIONAL CHANGES

As the antenna moves in elevation the position of the secondary will need to be adjusted to compensate for gravitational changes in the primary and support legs. The total movements, Δx_{tot} and Δz_{tot} will be on the order of a millimeter. Both of these will be sinusoidal functions of elevation so that the maximum rate of change will be 0.017 Δx_{tot} and 0.017 Δz_{tot} per degree of elevation change.

Over a 3-min interval between phase calibrations the elevation will change by at most 0.75° and the focus will change by 13 μm for Δx_{tot} or Δz_{tot} of 1 mm. The average change is half that. For changes between the source and the phase calibrator needing a 2° elevation change, the focus change will be ~35 μm. Over the 30 min between pointing and focus determinations the changes will be about ten times larger.

IV. Lateral Errors

Errors in the lateral positioning of the secondary, Δx or Δy produce coma and pointing offsets. The path error (and therefore the contribution to effective surface error) increases linearly with the offset. The contribution to the surface error is

$$\begin{aligned} \varepsilon_{eff} &= a_1 \Delta x \\ a_1 &= 19.1 \mu\text{m mm}^{-1} \end{aligned} \tag{1}$$

and the associated pointing change is

$$\Delta\theta_{sky} \approx 0.75 \frac{\Delta x}{f} \tag{2}$$

or about 20 µm for a thirtieth of a beam at λ 1-mm. Similar expressions apply to Δy. To adjust the lateral focus a gain curve can be made. This requires measuring the signal from a point source for different lateral positions of the secondary with the pointing offset corrected using (2). An alternative way is to measure the asymmetry of the first sidelobes which results from the coma. An imbalance of 1.3 dB results from an offset of $\Delta x = 0.1$ mm at $\lambda = 1$ mm, and this is associated with $\varepsilon_{eff} = 1.9$ µm.

V. Axial Errors

Axial errors, Δz, produce spherical aberration. The pathlength and surface error contributions are again in linear proportion to the displacement. For the nominal antenna parameters the contribution to the surface error is

$$\begin{aligned} \varepsilon_{eff} &= a_2 \Delta z \\ a_2 &= 80.7 \mu\text{m mm}^{-1} \end{aligned} \tag{3}$$

To determine the optimum focal setting, the secondary would be scanned in z and the signal from an astronomical point source measured. The half-power points are at about a half-wavelength on either side of the focus. We can estimate that if we measure the focus curve at $\Delta z \pm\lambda/2$, a relative accuracy of 2 % between the two points gives a measurement error for the focus position of ~0.02 λ. At 1-mm wavelength the error is therefore about 20 µm and (3) then gives $\varepsilon_{eff} = 1.6$ µm. Using the whole gain curve would give a better determination of the focus.

VI. Orthogonality of Axes

If the x- or y-axis is not perpendicular to the optical axis, or the z-axis is not parallel to it, the lateral and axial errors will be intermixed. The worst case is an x- or y-translation having a component along the optical axis. Since the z-focus is five times more sensitive than the x or y it would take an angular offset of ~1° in the x- or y-axis to have an effect at the 10 % level. The main effect would be a change in the shape of the gain curve (or measured beam shape). Around the nominal focus the symmetry of the measurement is not affected so the alignment precision is not reduced. Even for relatively large misalignments, only a few iterations of the x, y, and z-stages would be required to converge.

VII. Tilt Errors

The minimum aberrations occur for tilts around an axis through the prime focus. Rotations around any other axis can be decomposed into a rotation around the prime focus and a translation. The translation part is covered in section IV. A tilt of $\Delta\theta$ produces astigmatism with a contribution to the effective surface accuracy of

$$\begin{aligned} \varepsilon_{eff} &= a_3 \Delta\theta^2 \\ a_3 &= 8.16 \mu\text{m deg}^{-2} \end{aligned} \tag{4}$$

The reference axis defining zero tilt is the optical axis from the feed to the prime focus. This does not have to be coincident with the axis of the primary mirror paraboloid. It can be determined using a theodolite and level to look directly up at the prime focus (defined, say, by the holography feed) from the nominal receiver location. Finding the prime focus to within 10 mm defines the optical axis to better than 0.1° relative to gravity. Leveling the secondary mirror to this precision then gives a maximum error of 0.2° so that the contribution to surface error is less than 0.3 µm.

Note that a holographic measurement from the secondary focus would measure astigmatism due to the secondary tilt. It could then be removed by panel adjustment derived from the corresponding holography map.

VIII. Adjustment Range

To get a good focus determination a range of ± λ is required on all axes. At 30 GHz this means a total travel of 20 mm. Any allowances needed for manufacturing and assembly tolerances should be added to this.

IX. Discussion

All the errors due to focus setting as calculated above are added in quadrature giving an RSS (root sum of squares) of $\varepsilon_{eff} = 3.6$ µm. To achieve this the contributions of the translation stage encoders and drives must be negligible. The resolution and repeatability of the encoders should be a few times better than the required accuracy shown in Table II, or 5–10 µm.

TABLE II: REQUIRED SETTING ACCURACY OF SECONDARY MIRROR AND THE ASSOCIATED CONTRIBUTION TO THE SURFACE ERROR BUDGET

Parameter	Accuracy	Surface Error
Δx	50 µm	1.9 µm
Δy	50 µm	1.9 µm
Δz	20 µm	1.6 µm
$\Delta\theta_x$	0.2°	0.3 µm
$\Delta\theta_y$	0.2°	0.3 µm
	Total (RSS)	3.2 µm

Focus measurements will be made over a range of about $\pm\lambda$. At 3-mm wavelength this will impose a linearity requirement of about 0.2 % within this range.

If the z-focus is tracked during an observation it will also affect the phase. In order to know the pathlength to better than 5 µm the encoder would have to have an accuracy of 2.5 µm or better over the tracking range. Section III showed that the focus change over 3 min on the source is small enough to be ignored. The focus change going to the phase calibrator may be larger, but since the calibration is at a longer wavelength this focus adjustment can be omitted.

The intervals between focus and pointing calibrations occur on the longer interval of 30 min. For the maximum elevation change of 7.5° during this time, the focus could change by up to 130 µm. The secondary would need to be adjusted during this period at each phase calibration with a precision of ~10 µm. This also ensures that the contribution to the pointing error is less than 0.3 arcsec peak (<0.15 arcsec RMS)

The aberrations are functions of the focal length of the primary. Lateral aberrations vary as f^2, axial ones as $\sim f^2$, and tilts are proportional to f. Although the tolerances become smaller for shorter focal lengths, the focus curves are sharper and the relative estimation of the optimum focus remains about the same, independent of f

None of the measurements above can account for changes in the secondary position due to thermal changes or wind forces which occur between focus checks. A quadrant detector could be used to mitigate these variations. Existing systems meet the requirement of measuring to 10 µm or better.

The basic requirements for the secondary mirror mount are summarized in Table III. Note that these are maximum errors so the RMS errors will be about a factor of two less.

TABLE III: RECOMMENDED PARAMETERS FOR SECONDARY MIRROR FOCUS-TRANSLATION STAGE

Parameter	Value
Resolution in x, y	10 µm
Resolution in z	5 µm
Linearity in x, y	0.2 %
Linearity in z	0.2 %
Range in x, y	20 mm
Range in z	20 mm
Tilt of mirror relative to optical axis	0.2°
Orthogonallity of axes	1°