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ALMA Sensitivity, Supra-THz Windows, and 20 km baselines

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Introduction

This memo presents a calculation of the sensitivity of the ALMA array, given new thinking on number of antennas, resolutions of different configurations, and other considerations. Much of it is drawn from a similar calculation in Butler *et al.* (1999), which was patterned after Brown (1998). Numbers for the Supra-THz windows and baselines longer than 10 km are also presented and discussed.

Point Source Sensitivity

When making an image from interferometric array data, the flux density sensitivity, or rms noise in flux density units, can be written:

$$\Delta S = \frac{4\,\sqrt{2}\;k\;T_{sys}}{\gamma\,\epsilon_q\,\epsilon_a\,\pi\,D^2\,\sqrt{n_p[N(N-1)/2]\Delta\nu\Delta t}} \quad \mathrm{W/m^2/Hz}, \tag{1}$$

where T_{sys} is the system temperature, ϵ_a is the aperture efficiency, ϵ_q is the correlator quantization efficiency, D is the antenna diameter, n_p is the number of simultaneously sampled polarizations, N is the number of antennas in the array, $\Delta\nu$ is the bandwidth, Δt is the integration time, and γ is a gridding parameter that we set equal to unity. Assume that $N = 64; n_p = 2; D = 12;$, and $\epsilon_q = 0.95$. Then equation 1 simplifies to:

$$\Delta S = \frac{290\;T_{sys}}{\epsilon_a\,\sqrt{\Delta\nu\Delta t}} \quad \mathrm{mJy}. \tag{2}$$

The aperture efficiency is:

$$\epsilon_a = \epsilon_0\,e^{-\left(\frac{4\,\pi\,\sigma}{\lambda}\right)^2} \quad , \tag{3}$$

which for the goals of ALMA ($\sigma = 25\,\mu$m, $\epsilon_0 = 0.80$) takes the values shown in Table 1 over the possible range of frequencies of operation for ALMA. Note that we add the supra-THz windows because even though they are not currentlya part of ALMA proper, it is interesting to see if anything useful can be done with ALMA at these frequencies.

Table 1: Aperture efficiency of the ALMA antennas.

Frequency (GHz)	Wavelength (μm)	ϵ_a
35	8600	0.80
110	2700	0.79
230	1300	0.75
345	870	0.70
409	650	0.63
675	440	0.48
850	350	0.36
1020	290	0.25
1350	220	0.10
1500	200	0.07

Refer T_{sys} to a point outside the terrestial atmosphere and compute it as:

$$T_{sys} = T'_{rx}\, e^{\tau_0 A} + \epsilon_l\, T'_{atm} \left(e^{\tau_0 A} - 1\right) + (1 - \epsilon_l)\, T'_{sbr}\, e^{\tau_0 A} + T'_{cmb} \quad , \tag{4}$$

where T'_{rx} is the receiver temperature, τ_0 is the opacity of the atmosphere at zenith, A is the airmass, ϵ_l is the fraction of the antenna power that is received in the forward direction (i.e., the fraction that is on the sky in the main lobe and all the forward sidelobes), T'_{atm} is the effective atmospheric temperature, T'_{sbr} is the temperature onto which the spillover falls, and T'_{cmb} is the cosmic microwave background temperature. The terms of T_{sys} represent the contributions from the receiver, the sky, the "antenna", and the CMB. The primes on the temperatures indicate that they are effective radiation temperatures, and should be calculated with a Planck correction (Ulich & Haas 1976):

$$T'_x = \frac{h\nu/k}{e^{h\nu/kT_x} - 1} \quad , \tag{5}$$

where ν is the frequency, and T_x is the physical temperature.

Assume that $T_{sbr} = T_{amb}$ where T_{amb} is the ambient surface temperature. Assume further that $T_{atm} = 70.2 + 0.72\ T_{amb}$ (Bevis *et al.* 1992), which has been verified to be a fairly accurate representation of the effective atmospheric temperature by comparison to detailed atmospheric emission models. Assume $T_{amb} = 269$ K, the average surface temperature at the ALMA site.

The ALMA receivers are image separating receivers (SSB) with the unwanted sideband terminated at 4 K. The noise temperature of these receivers can be written:

$$T_{rx}(\nu) = \alpha(\nu)\,\frac{h\,\nu}{k} + 4 \quad , \tag{6}$$

where α is a photon limit multiplier (the photon limit for receivers is $h\nu/2k$ - see Kerr *et al.* 1996). Current receiver technology provides that $\alpha(\nu) \lesssim 3$ for $\nu < 500$ GHz, $\alpha(\nu) \sim 6$ for 675 GHz, and $\alpha(\nu) \sim 12$ for 850 GHz (taken from the presentation of T. Noguchi at the ALMA meeting prior to the 1999 URSI GA, but see also the papers in session JDC from that meeting). We take the current technology value of $\alpha(\nu) = 3$ for $\nu < 500$ GHz, and feel that it is reasonable to assume values of $\alpha(675) = 4$ and $\alpha(850) = 8$. It is unclear what values are reasonable to assume for the supra-THz window frequencies, given the rapidly changing mixer technology, but we assume values of $\alpha(1040) = 15, \alpha(1350) = 20$, and $\alpha(1500) = 25$.

Table 2: Estimated T_{sys} for ALMA at elevation=50°.

Frequency (GHz)	τ_0	$T'_{rx}\,e^{\tau_0 A}$ (K)	$\epsilon_l\,T_{atm}\,(e^{\tau_0 A}-1)$ (K)	$(1-\epsilon_l)\,T_{sbr}\,e^{\tau_0 A}$ (K)	T_{sys} (K)
35^+	0.016	8.4	5.1	13.7	29
110^+	0.049	18.5	16.5	14.2	50
230^+	0.078	35.3	26.1	14.6	76
345^+	0.276	65.6	105	18.7	190
409^+	0.544	110	250	26.3	380
675^+	1.789	1200	2200	129.5	3500
850^+	1.601	2500	1600	99.9	4200
675^*	0.456	210	190	22.9	430
850^*	0.437	550	180	22.0	750
1020^*	1.876	8200	2400	140.5	10700
1350^*	1.741	12200	1900	114.4	14200
1500^*	1.713	16400	1800	108.8	18300

$^+$ PWV = 1.5 mm. * PWV = 0.35 mm.

For the second two terms we adopt the ALMA antenna goal of $\epsilon_l = 0.95$, i.e., 95% of the received power comes from the forward direction. We will compute T_{sys} at an airmass of 1.3 (50° elevation) and use for the frequency dependent optical depths on the Chajnantor site the opacities produced by a model atmosphere for that site. These model opacities are taken from the Liebe model (Liebe 1989) for frequencies $<$ 1000 GHz, and from the Traub & Stier model (as implemented by Grossman's AT - but see Traub & Stier [1976] for a description of the model) for the higher frequencies. For the frequencies $<$ 1000 GHz, the nominal model contains 1.5 mm of precipitable water vapor (PWV), which is roughly the median at the site over all hours and seasons. The supra-THz windows are entirely opaque at this PWV, so a model atmosphere with less PWV is used to investigate those frequency windows. The PWV selected is 0.35 mm. The values for the 675 and 850 GHz windows are also calculated for this PWV, for comparison. This value of PWV (0.35 mm) is achieved under good conditions at the site, but not the best conditions. As an illustration, the median PWV in August 1999 was 0.4 mm (personal communication, A. Otarola), and over the 4.5 years of site testing data, the 225 GHz tipper results indicate that conditions are this good about 5% of the time. The opacities (and ratios) produced by this model agree well with those measured with FTS devices at or near the ALMA site (Matsuo *et al.* 1998; Matsushita *et al.* (1999); Paine & Blundell 1999).

The terms in the T_{sys} equation above, along with the resultant T_{sys} are shown in Table 2. These numbers agree relatively well with those of Jewell and Mangum (1997), Brown (1998), and Butler *et al.* (1999) when a common set of assumptions is used.

Continuum sensitivity

The continuum bandwidth for ALMA is 8 GHz per polarization, so assign $\Delta\nu = 8$ GHz. Using the system temperatures in Table 2, the aperture efficiencies in Table 1, and an integration time of 1 minute, the sensitivities shown in Table 3 are derived.

Table 3: Continuum sensitivity for ALMA in 1 minute.

Frequency (GHz)	ΔS_{cont} (mJy)
35^+	0.015
110^+	0.026
230^+	0.042
345^+	0.11
409^+	0.24
675^+	3.0
850^+	4.8
675^*	0.36
850^*	0.85
1020^*	17
1350^*	54
1500^*	110

$^+$ PWV = 1.5 mm. * PWV = 0.35 mm.

Spectral line sensitivity

For spectroscopic observations we use a velocity channel width Δv and write $\Delta\nu = \nu\Delta v/c$, leaving:

$$\Delta S = \frac{5.0\ T_{sys}}{\epsilon_a \sqrt{\nu_{GHz}\Delta v_{km/s}\Delta t}} \quad \text{mJy}, \tag{7}$$

where ν_{GHz} is the frequency in GHz, and $\Delta v_{km/s}$ is the velocity channel width in km/s. Using the system temperatures in Table 2, the aperture efficiencies in Table 1, an integration time of 1 minute, and a velocity channel width of 1 km/s, the sensitivities shown in Table 4 are derived.

Table 4: Spectral line sensitivity for ALMA in 1 minute in a 1 km/s channel.

Frequency (GHz)	ΔS_{line} (mJy)
35^{+}	3.9
110^{+}	3.9
230^{+}	4.3
345^{+}	9.3
409^{+}	18
675^{+}	180
850^{+}	260
675^{*}	22
850^{*}	45
1020^{*}	840
1350^{*}	2300
1500^{*}	4500

$^{+}$ PWV = 1.5 mm. * PWV = 0.35 mm.

Brightness temperature sensitivity

Consider an observation of a source which fills the synthesized beam, and assume that the source intensity is large enough that it is in the Rayleigh-Jeans portion of the spectrum (so that no Planck correction is necessary). In this case, the brightness temperature sensitivity is given by:

$$\Delta T = \frac{\Delta S\ \lambda^2}{2\ k\ \Omega_s} \quad , \tag{8}$$

where λ is the wavelength, and Ω_s is the synthesized beam solid angle. For an image which is restored with a circular gaussian of width θ_s (e.g., the result of CLEAN or relatives), this solid angle is given by:

$$\Omega_s = \frac{\pi}{4\ \ln 2}\ \theta_s^2 \sim \frac{\pi}{4\ \ln 2}\ \frac{\lambda^2}{B_{max}^2} \quad , \tag{9}$$

for maximum physical baseline length B_{max}. Substituting this into the equation for brightness temperature sensitivity yields:

$$\Delta T = \frac{2\ \ln 2}{\pi\ k}\ B_{max}^2\ \Delta S = 0.32\ B_{max_{km}}^2\ \Delta S_{mJy} \quad , \tag{10}$$

where $B_{max_{km}}$ is in km, and ΔS_{mJy} is in mJy. We assume configurations for ALMA with $B_{max_{km}} = 0.2$, 0.4, 1.0, 3.0, 10.0, and 20.0. Using the values for noise flux density in Tables 3 and 4 yields the brightness temperature sensitivities in Table 5.

Table 5: Brightness temperature sensitivity for ALMA in 1 minute.

	B_{max} = 0.2 km		0.4 km		1 km		3 km		10 km		20 km	
frequency (GHz)	ΔT_{cont} (K)	ΔT_{line} (K)	ΔT_{cont} (K)	ΔT_{line} (K)	ΔT_{cont} (K)	ΔT_{line} (K)	ΔT_{cont} (K)	ΔT_{line} (K)	ΔT_{cont} (K)	ΔT_{line} (K)	ΔT_{cont} (K)	ΔT_{line} (K)
35^+	0.0002	0.050	0.0008	0.20	0.0048	1.3	0.043	11	0.48	130	1.9	500
110^+	0.0003	0.049	0.0013	0.20	0.0084	1.2	0.075	11	0.84	120	3.3	490
230^+	0.0005	0.054	0.0021	0.22	0.013	1.4	0.12	12	1.3	140	5.3	540
345^+	0.0014	0.12	0.0057	0.48	0.036	3.0	0.32	27	3.6	300	14	1200
409^+	0.0030	0.23	0.012	0.93	0.076	5.8	0.68	52	7.6	580	30	2300
675^+	0.038	2.3	0.15	9.1	0.96	57	8.6	510	96	5700	380	23000
850^+	0.062	3.3	0.25	13	1.5	82	14	740	150	8200	610	33000
675^*	0.0046	0.28	0.019	1.1	0.12	6.9	1.0	62	12	690	46	2800
850^*	0.011	0.58	0.044	2.3	0.27	14	2.5	130	27	1400	110	5800
1020^*	0.22	11	0.89	43	5.5	270	50	2400	550	27000	2200	110000
1350^*	0.69	29	2.8	120	17	730	160	6600	1700	73000	7000	300000
1500^*	1.4	57	5.7	230	36	1400	320	13000	3600	140000	14000	570000

$^+$ PWV = 1.5 mm. * PWV = 0.35 mm.

Discussion

Supra-THz windows

It seems clear from the above sensitivities that we should attempt, if possible, to put receivers in the 1020, 1350, and 1500 GHz windows on the ALMA antennas. There is no other instrument in existence, or currently being planned at an advanced stage, which comes close to the sensitivities shown above. Given the amount of time that this good sensitivity can be obtained at the ALMA site, it seems short-sighted to *not* include receivers in these windows. If it turns out to be relatively simple and inexpensive to include the ability to add in receivers for these windows later in the project (e.g., by explicitly allowing space in the dewar), then we should certainly do so. Note in addition that if the actual delivered antennas have a surface accuracy which meets the design goal (20 μm) rather than the specification (25 μm), then the efficiencies get better by about a factor of two, implying a similar improvement in sensitivity. If all of the antennas do not perform well at the higher frequencies, we might consider equipping only a subset of the best antennas with receivers in these windows. One obvious drawback is the very small field of view of our 12 meter diameter antennas at these high frequencies. A possible solution to this is utilization of an array of smaller antennas to observe above 1 THz. This would alleviate the short-spacing problem at the lower frequencies as well.

20 km baselines

It also seems clear from the brightness sensitivity shown above that going to 20 km baselines is not as silly an idea as it once seemed. The brightness sensitivity reached ($\Delta T_{cont} \lesssim 10$ K in 1 minute for $\lambda \gtrsim 1$ mm) is a useful one for investigating phenomena at very high resolution (note that the resolution of a 20 km array at 345 GHz is $\sim$ 10 masec!). Here, in contrast to the case for the Supra-THz window receivers, there are some more serious technological, engineering, and cost issues. In addition, preliminary studies by Lee Mundy and collaborators indicate that even in the 10 km array the sampling of the UV-plane is poor, resulting in noticably poorer imaging performance than is achieved with the 3 km array. This needs to be more fully understood (e.g., can we design a 20 km array that, when supplemented with data from the 10 and 3 km arrays gives good UV-coverage?). It is also not clear that we could fit a 20 km array on the site, given current topography and geopolitical constraints. A detailed analysis of these issues should be completed before a strong recommendation to have these longer baselines is issued.

Caveats

We note two considerations concerning the numbers presented above. First, we have assumed here that we have an almost perfect instrument, i.e., there is no extra phase fluctuation from electronics (other than thermal noise), atmosphere, pointing errors, etc... In particular, for the continuum sensitivity, we have assumed that the 1/f noise of the receivers does not dominate the thermal noise. This is the current specification for that quantity, and places a strong constraint on the 1/f noise of the receivers, as has been recognized. Secondly, the numbers presented here are only really valid for unresolved or partially resolved sources. For complex resolved sources (perhaps a large fraction of sources which will be observed by ALMA), it is not the strict sensitivity that is important, but rather the imaging *fidelity* which will determine the quality of many of the measurements/images. Simulations are indicating that the achievable fidelity might not be better than a few percent (Holdaway 1997; Wright 1999). Work continues on means of improving image fidelity to match the unparalleled raw sensitivity of the instrument.

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