Roughness in the Mars Emission Model

The roughness of a planetary surface will modify the outgoing thermal emission from the subsurface in several ways. What we are concerned with here is the modification of the effective emissivity of the surface. This is described in many references, including Heiles & Drake (1963), Hagfors & Moriello (1965), Cuzzi (1974), Golden (1979), and Mitchell & de Pater (1994).

Assume we have an antenna measuring the thermal emission which is sensitive to a single linear polarization, and that the antenna (or its feed) can be rotated to any angle. Then write the response to two orthogonal directions as:

$$T_{\|} = T \ \left(E_{\|} \cos^2\phi + E_{\perp} \sin^2\phi\right) \tag{1}$$

and

$$T_{\perp} = T \ \left(E_{\perp} \cos^2\phi + E_{\|} \sin^2\phi\right) \tag{2}$$

where T is the physical temperature, ϕ is the angle between the orientation of the linear feed and the location on the planetary surface, and $E_{\|}, E_{\perp}$ are the surface emissivities in the two directions. The emissivities are given as a function of the emission angle (angle between surface normal and observing direction), θ, by:

$$E_{\|}(\theta) = 1 - R_{\|}(\theta) \tag{3}$$

and

$$E_{\perp}(\theta) = 1 - R_{\perp}(\theta) \tag{4}$$

where $R_{\|}, R_{\perp}$ are the Fresnel reflectivities in the two directions. These can be written

$$R_{\|}(\theta) = \left|\frac{\epsilon\,\cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\epsilon\,\cos\theta + \sqrt{\epsilon - \sin^2\theta}}\right|^2 \tag{5}$$

and

$$R_{\perp}(\theta) = \left|\frac{\cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon - \sin^2\theta}}\right|^2 \tag{6}$$

for a surface with dielectric constant ϵ. If the polarization is not of concern (either the two linear polarizations are averaged together, or circular polarization is measured), then the angle ϕ falls out when averaging over the two linear polarizations, and we can write the average emissivity as

$$E(\theta) = \frac{E_{\|}(\theta) + E_{\perp}(\theta)}{2} = 1 - \frac{R_{\|}(\theta) + R_{\perp}(\theta)}{2} \tag{7}$$

(Of course if we are concerned with the difference in the linear polarizations then we can retain the separation of the two emissivities.)

The surface roughness is modelled by assuming that the surface is constructed of many facets, each of which is much smaller than the resolution element on the planet, but much larger than the wavelength being observed. Each of these facets has an orientation relative to the mean surface given by two angles - the angle between the surface normal and the mean surface normal (the y-axis in the standard right-handed x-y-z coordinate system), ψ, and the angle between the projection of the surface normal on the x-z plane and the x-axis, δ. See Figure 1 for the coordinate system and facet normal geometry.

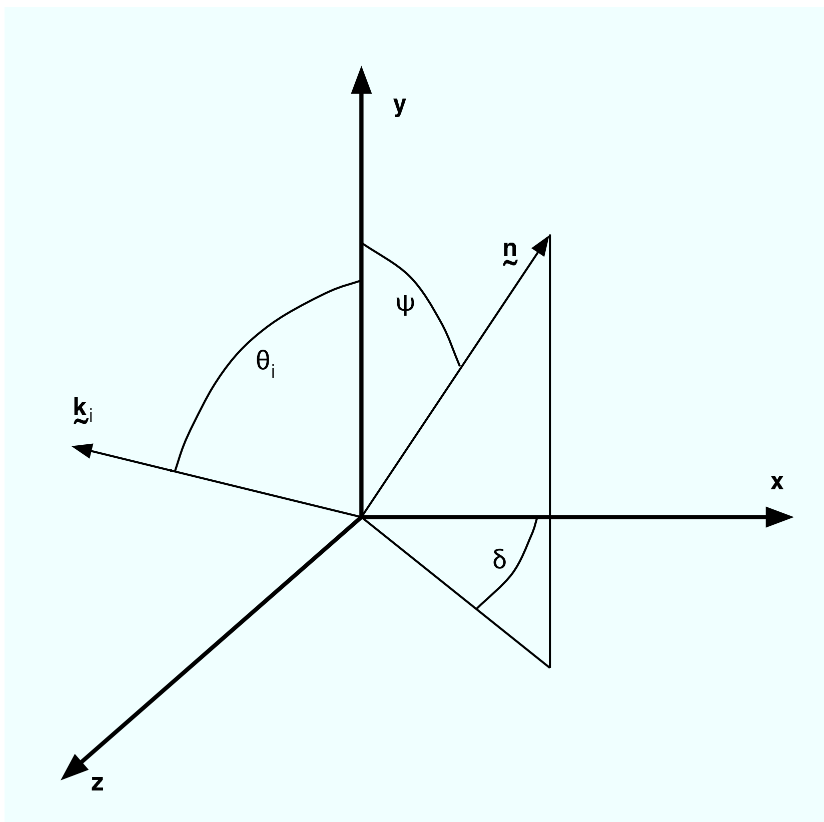

Figure 1: Geometry for roughness calculations.

For the random facets, the angle δ is evenly distributed over the range $[0,2\pi)$, while the angle ψ is dependent upon the statistical distribution of slopes (more on that later). To construct a single facet, we therefore need simply generate two random numbers and derive these two angles from them. Using these two angles, we have the definition of a planar facet, with normal in direction $\mathbf{n}$, given by

$$\mathbf{n} = (n_x, n_y, n_z) = (\sin\psi\ \cos\delta, \cos\psi, \sin\psi\ \sin\delta) \tag{8}$$

Noting Figure 1 again, the mean surface normal is the unit vector along the y-axis. Take the mean emission to be along a direction $\mathbf{k_i}$ in the y-z plane at an angle θ_i from the y-axis

$$\mathbf{k_i} = (k_{i_x}, k_{i_y}, k_{i_z}) = (0, \cos\theta_i, \sin\theta_i) \tag{9}$$

then the emission angle for a given facet is just the angle between that direction and the normal for that facet:

$$\cos\theta_e = \mathbf{n}\cdot\mathbf{k_i} = \cos\psi\ \cos\theta_i + \sin\psi\ \sin\delta\ \sin\theta_i \tag{10}$$

This value for θ_e can then be used in the equations above for the emissivity as a function of angle.

For each facet, the deviation of the normal from the mean surface normal (the angle ψ) can be calculated given probability distribution of slopes of the surface. The roughness model allows for either of two distributions, both developed in Muhleman (1964). The first is a Gaussian distribution of surface heights and a Rayleigh distribution of surface lengths (called the "Gaussian" model), which results in a probability distribution of slopes of

$$p'(\psi)d\psi = \frac{1}{2}\ \alpha'^2\ \frac{\cos\psi d\psi}{\left(\sin^2\psi + \alpha'^2\cos^2\psi\right)^{3/2}} \tag{11}$$

where the parameter α' is the mean surface slope. The second is an exponential distribution of heights and Poisson distribution of surface lengths (called the "Exponential" model), which results in a probability distribution of slopes of

$$p(\psi)d\psi = 2\ \alpha^2\ \frac{\cos\psi d\psi}{(\sin\psi + \alpha\cos\psi)^3} \tag{12}$$

where once again the parameter α is the mean surface slope.

To convert these probability distribution functions (PDFs) to cumulative distribution functions (CDFs), integrate over angle. First, for the Gaussian case

$$F'(\psi) = \frac{1}{2}\,\alpha'^2 \int\limits_{-\pi/2}^{\psi} \frac{\cos\psi}{\left(\sin^2\psi + \alpha'^2\cos^2\psi\right)^{3/2}}\,d\psi = \frac{1}{2}\left(1 + \frac{\tan\psi}{\sqrt{\tan^2\psi + \alpha'^2}}\right) \tag{13}$$

and for the Exponential case

$$F(\psi) = 2\,\alpha^2 \int\limits_{0}^{\psi} \frac{\cos\psi}{\left(\sin\psi + \alpha\cos\psi\right)^3}\,d\psi = 1 - \frac{\alpha^2}{\left(\tan\psi + \alpha\right)^2} \tag{14}$$

To draw a random angle from one of these CDFs, set it equal to a uniform deviate, ζ (such as can be calculated by a random number generator), then invert to solve for the angle. For the Gaussian case

$$\psi = \tan^{-1}\left(b\;\alpha\,\sqrt{\frac{1}{1-b^2}}\right) \tag{15}$$

where $b = 2\,\zeta - 1$. For the Exponential case

$$\psi = \tan^{-1}\left[\alpha'\left(\sqrt{\frac{1}{1-\zeta}} - 1\right)\right] \tag{16}$$

So, for each mean emissivity angle (θ_i) the model needs a roughness-modified emissivity for ($\langle\, E(\theta_i)\,\rangle$), calculate the emissivity of N random facets (the model uses 10000) by generating two random numbers for each facet and deriving the two angles δ and ψ from them. Then calculate the effective emission angle and emissivity of the facet, θ_{e_j} and E_j. Then the roughness-modified average emissivity is

$$\langle\, E(\theta_i)\,\rangle \;=\; \frac{\sum\limits_{j=1}^{N} E_j\cos\theta_{e_j}}{\sum\limits_{j=1}^{N}\cos\theta_{e_j}} \tag{17}$$

One detail of note remains. Near the limb some facets will actually not be visible, notably for large mean surface slopes. The model ignores such facets. This is similar to the "clipped cosine" in the Mitchell & de Pater (1994) model.

References

Cuzzi 1974, AJ, 189, 577
Golden 1979, Icarus, 38, 451
Hagfors & Moriello 1965, JRNBS, 69D, 614
Heiles & Drake 1963, Icarus, 2, 281
Mitchell & de Pater 1994, Icarus, 110, 2
Muhleman 1964, AJ, 69, 34