Notes on Axis Intersection for MMA Antennas

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1 Introduction

This memo incorporates some notes I wrote down on 1998-Dec-10 when Peter Napier asked me to look into the problem of specifying the axis intersection quantities (two angles and a separation) for the MMA antennas. I provide no further information than what is directly in the notes, but it seemed that the information therein should be retained historically, so I've made this electronic version.

Some notes on azimuth & elevation axis non-intersection for MMA

much of this is based on Cam Wade's memo (VLA Test Memo # 104)

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first, what if angles are not precisely 90°? I've drawn on Cam's figure 1 the angles that we are specifying for MMA antennas (θ_1 & θ_2).

If $\theta_2 \neq 90°$, this simply mimics an azimuth pointing offset, and can be handled as such, I think.

If $\theta_1 \neq 90°$, this changes the vector $\underset{\sim}{D}$ which connects P to P' (see figure). If we let $\theta_1 = 90° - \Psi$, and assume Ψ is small so that $\cos\Psi \sim 1$ and $\sin\Psi \sim \Psi$, then,

$$\underset{\sim}{D} \cong \begin{bmatrix} (a - b\sin h)\cos Z + (c - b\Psi)\sin Z \\ -(c - b\Psi)\cos Z + (a - b\sin h)\sin Z \\ c\Psi + b\cosh \end{bmatrix}$$

this adds some terms (involving $b\Psi$ & $c\Psi$) to the fringe phase error:

$$\Delta\phi = \alpha\cosh - \Delta h(\alpha\sinh - \beta) - \gamma\Delta Z\cosh + \Delta\phi_\theta$$

the $\Delta\phi_\theta$ are the additional terms:

$$\Delta\phi_\theta = \beta\Psi\Delta h\cosh + \gamma\Psi(\sinh + \Delta h\cosh)$$

$$= \Psi\Delta h\cosh(\beta + \gamma) + \gamma\Psi\sinh$$

assuming Ψ is small, $\Delta\phi_\theta$ is negligibly small

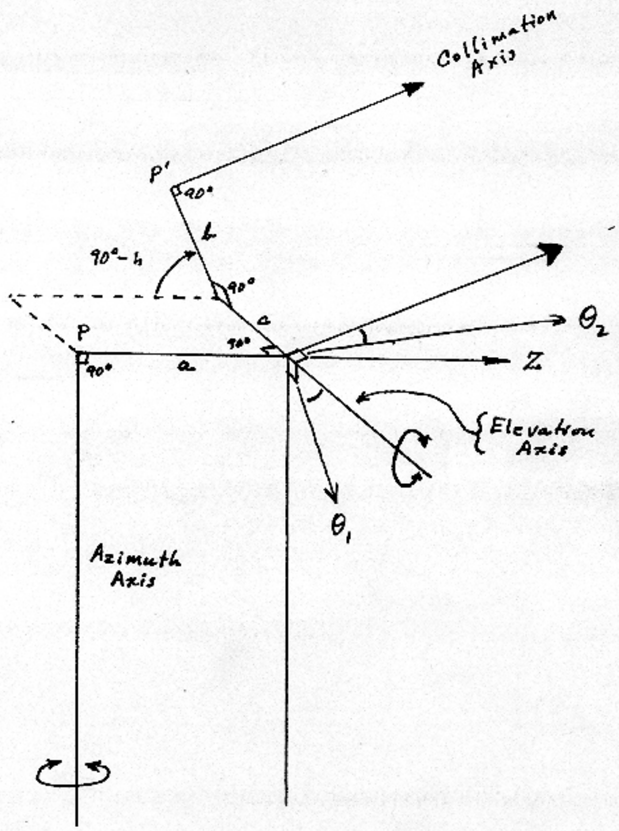

Figure 1 from Wade 1974

Now, what if we make no attempt to measure a, b, and c on the antennas, and rely on our secondary calibrator to account for $\Delta\phi$?

in the simplest case, assume that at $t=0$, we measure phase on a calibrator at elevation h_{c_0}, then after some interval Δt, we measure it again (at elevation h_{c_1}). if we use simple linear interpolation of these measured phases, and apply them to the source visibilities, we get an error like:

$$\Delta\phi_k(t) = \alpha \cos[h_s(t)] - \phi_e(t)$$

$$= \alpha \left\{ \cos[h_s(t)] - \left[\cos h_{c_0} + \frac{\cos h_{c_1} - \cos h_{c_0}}{\Delta t} \cdot t \right] \right\}$$

i.e., there is some error due to the calibrator not being at the same elevation as the source, and some due to approximating a cosine by a linear interpolation.

as long as Δt is relatively small ($\lesssim$ 30 mins) the error due to the different source and calibrator elevations dominates. in this case,

unless the variation of elevation due to time causes phase to go through $\gtrsim \frac{1}{4}$ turn. (so that you can't interpolate ϕ_e correctly). 30 min → 7.5 deg, so about same constraint (of same order anyway) as derived on next page...

$$|\Delta\phi_k|_{max} \lesssim \alpha\, \delta h \quad \sin h_{max}$$

where δh is the elevation difference between source & calibrator.

what to use for δh?

based on Bob Brown's noise temps (in his SPIE paper), and Scott Foster's derived distances to calibrators at 90 GHz (MMA memo 124), and a scaling for source number counts which goes like:

$N_\nu \propto S_\nu^{-1.5} \nu^{3.5}$

(see Kitayama et al. for S_ν scaling, and Franceschini et al. for ν scaling)

and assume you need 10σ per baseline in a 1 min calibration scan, then you might have the following:

ν (GHz)	δh (deg)
115	1.5
230	0.6
345	0.4
410	0.4
675	0.6

this is for 90% of sources...

these numbers seem a bit small. let's assume that δh might be as large as 3 deg. let us also assume that we want $|\Delta\phi_k|$ to be less than 20° (1/18 turns). then:

note that this fundamentally limits astrometry... (this error, if left unaccouted for)

$$\frac{1}{18} \gtrsim \alpha \cdot 0.052$$

$$\Rightarrow \alpha \leq 1.1$$

at MMA's shortest wavelength (850 GHz) - $\lambda \sim 350\ \mu m$, so

$\boxed{\text{accuracy of} \quad a \lesssim 200\ \mu m}$

REMEMBER THAT THIS IS ONLY NEEDED IF $\alpha \cos h$ IS NOT MEASURED/CALIBRATED what if we measure/calibrate the $\alpha \cos h$ term, but ignore the rest?

NOTE: this is what the VLA does - this is the so-called k-term correction. It is measured every year or so, when in C or D-config, and is usually quite stable over time. The measurement of α (a) is made possible by being able to go OTT. This makes α (a) easily distinguishable from other baseline/pointing terms.

in this case, the residual phase is:

$$\Delta\phi'' = -\alpha \, \Delta h \sin h + \beta \Delta h - \gamma \Delta z \cos h$$

use this to constrain a, b & c separately...

$$\Delta\phi''_a = -\alpha \, \Delta h \sin h$$

let $h \to 90°$, $\Rightarrow \sin h \to 1$

$$|\Delta\phi''_a| = \alpha \Delta h$$

$\alpha = a/\lambda$, and let Δh be some multiple of HPBW ($\Delta h = n\lambda/D$), then

$$|\Delta\phi''_a| \sim an/D$$

let $D = 10$ m and $|\Delta\phi''_a| \lesssim 20°$ ($\frac{1}{18}$ turns, $\frac{\pi}{9}$ rad) and $n \sim 10$, then

$\boxed{\text{accuracy of } a \lesssim 3 \text{ cm}}$

Similar constraint for b & c, so this is unimportant... (relatively) bjb 10 Dec 1998

what about decorrelation due to $\alpha \cos h$ term?

even if you correct for $\alpha \cos h$, you have to worry about how much it changes over an integration cycle. the change in $\alpha \cos h$ over 1 integration is roughly:

$$\Delta(\alpha \cos h) \sim \alpha \Delta h$$

with the max elevation change given by:

$$\Delta h \sim \frac{2\pi \, \Delta t}{86400} \quad \text{rad}$$

for integration time Δt sec. if we want the error over an integration cycle to be less than 20 deg, then,

$$\alpha \times \frac{2\pi \, \Delta t}{86400} \lesssim \frac{20}{360}$$

$$\Rightarrow \alpha \lesssim \frac{760}{\Delta t} \quad \text{turns}$$

at 350 μm, this corresponds to:

$$\boxed{\text{accuracy of } a \lesssim \frac{130}{\Delta t} \text{ mm}}$$

e.g. @ $\Delta t = 10$ sec, accuracy of $a \lesssim 1.3$ cm

if this is not satisfied, given eventual values for a and Δt, then we may need a 2nd order $\alpha \cos h$ correction (provide $\alpha \cos h$ and $\frac{\partial}{\partial t} \alpha \cos h$ to fringe frequency calculator - this is how VLA-PT will be done).

[other refns: Wade, ApJ, 162, 381
Sovers et al., Rev. Mod. Phys, 70, 1393]